Lecture notes: 02

Rectifier – Half wave rectifier and Full wave rectifier

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Contents

1 Alternating current (AC)

Alternating current (AC) describes the flow of charge that changes direction periodically. As a result, the voltage level also reverses along with the current. AC is used to deliver power to houses, office buildings, etc. The AC voltage of a periodic waveform may be written as

$$V(t) = V_m \sin(\omega t), \tag{1}$$

where $\omega = 2\pi/T$ is the angular frequency of the waveform or voltage and T is the time period of the voltage.

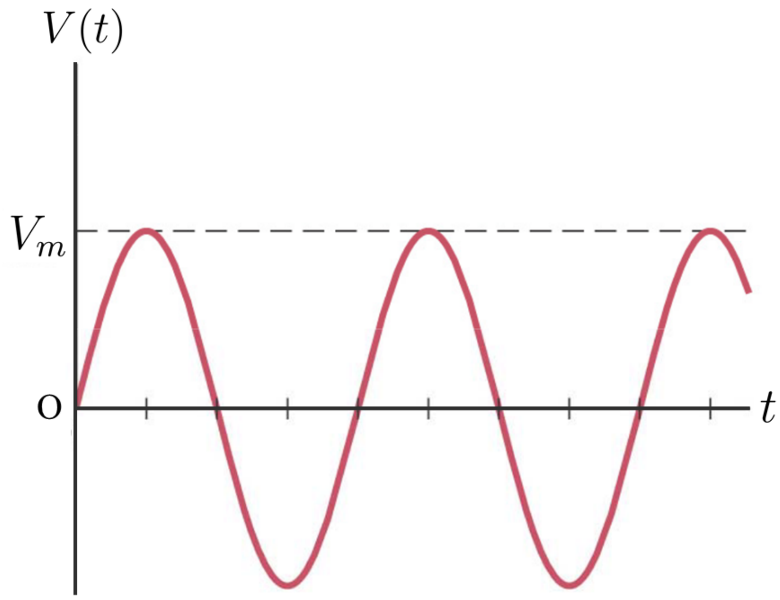

Figure 1: AC voltage $V(t)$

Average voltage over the time period

The average value of $V(t)$ over the time period T is defined as

$$\boxed{\overline{V} = \frac{1}{T}\int_0^T V(t)\mathrm{d}t.} \tag{2}$$

Hence

$$\begin{aligned}
\overline{V} &= \frac{V_m}{T}\int_0^T \sin(\omega t)\mathrm{d}t \\
&= \frac{V_m}{T}\left[-\frac{\cos(\omega t)}{\omega}\right]_0^T \\
&= \frac{V_m}{\omega T}\{-\cos(\omega T) + \cos 0\} \\
&= \frac{V_m}{2\pi}\{-\cos(2\pi) + \cos 0\} \\
&= \frac{V_m}{2\pi}(-1+1) \\
&= 0.
\end{aligned} \tag{3}$$

Therefor, the average value of an AC voltage over the time period of the oscillation is zero.

Average voltage over half of the time period

Since the average value of the AC voltage over the time period is zero, we may calculate the average value over the half of the time period using a similar definition as (2). Therefore

$$\begin{aligned}
V_{\text{avg}} &= \frac{1}{T/2}\int_0^{T/2} V(t)\mathrm{d}t \\
&= \frac{2V_m}{T}\int_0^{T/2} \sin(\omega t)\mathrm{d}t \\
&= \frac{2V_m}{T}\left[-\frac{\cos(\omega t)}{\omega}\right]_0^{T/2} \\
&= \frac{2V_m}{\omega T}\{-\cos(\omega T/2)+\cos 0\} \\
&= \frac{2V_m}{2\pi}\{-\cos(\pi)+\cos 0\} \\
&= \frac{V_m}{\pi}(+1+1) \\
&= \frac{2}{\pi}V_m \\
&\approx 0.637\,V_m. \qquad (4)
\end{aligned}$$

The RMS value of the AC voltage

The term "RMS" stands for "Root-Mean-Squared", also called the effective or heating value of alternating current, is equivalent to a DC voltage that would provide the same amount of heat generation in a resistor as the AC voltage would if applied to that same resistor.

RMS is not an "Average" voltage, and its mathematical relationship to peak voltage varies depending on the type of waveform. The RMS value is the square root of the mean (average) value of the squared function of the instantaneous values. For the voltage $V(t)$ given in (1) it can be written as

$$\boxed{V_{\text{rms}} = \left[\frac{1}{T}\int_0^T V^2(t)\mathrm{d}t\right]^{1/2}.} \qquad (5)$$

Hence

$$
\begin{aligned}
V_{\text{rms}}^2 &= \frac{V_m^2}{T}\int_0^T \sin^2(\omega t)\mathrm{d}t \\
&= \frac{V_m^2}{2T}\int_0^T 2\sin^2(\omega t)\mathrm{d}t \\
&= \frac{V_m^2}{2T}\int_0^T \{1-\cos(2\omega t)\}\,\mathrm{d}t \\
&= \frac{V_m^2}{2T}\int_0^T \mathrm{d}t - \frac{V_m^2}{T}\int_0^T \cos(2\omega t)\mathrm{d}t \\
&= \frac{V_m^2}{2T}\Big[T\Big]_0^T - \frac{V_m^2}{2T}\left[\frac{\sin(2\omega t)}{2\omega}\right]_0^T \\
&= \frac{V_m^2}{2} - \frac{V_m^2}{4\omega T}\Big\{\sin(2\omega T)-\sin(0)\Big\} \\
&= \frac{V_m^2}{2} - \frac{V_m^2}{4\omega T}\Big\{\sin(4\pi)-\sin(0)\Big\} \\
&= \frac{V_m^2}{2} - \frac{V_m^2}{4\omega T}(0-0) \\
&= \frac{V_m^2}{2}. \qquad (6)
\end{aligned}
$$

Therefore,

$$\boxed{V_{\text{rms}} = \frac{V_m}{\sqrt{2}} \approx 0.707\,V_m.} \qquad (7)$$

Important relations to remember

A few handy things to keep in mind about RMS values that apply when dealing with a sine wave, are as follows:

- $V_{\text{rms}} \approx 0.707\times$ peak AC voltage = 70.7 % of peak voltage
- Peak AC voltage $\approx 1.414\ \times V_{\text{rms}}$ = 141.1 % of V_{rms}
- $V_{\text{avg}} \approx 0.637\times$ peak AC voltage = 63.7 % of peak voltage
- $\dfrac{V_{\text{rms}}}{V_{\text{avg}}} = \dfrac{\pi}{2\sqrt{2}} \approx 1.11$

2 Half wave rectifier

A half wave rectifier is a type of rectifier that only allows one half-cycle of an AC voltage waveform to pass, blocking the other half-cycle. Half-wave rectifiers are used to convert

AC voltage to DC voltage, and only require a single diode to construct. A half wave rectifier is the simplest form of rectifier available.

Figure 2 shows the input AC voltage waveform, the circuit diagram and the final output voltage waveform of a half wave rectifier. During the positive half cycle, the diode is forward biased making the current flow through the load resistor. While during the Negative half cycle the diode is reverse biased so it stops the current flow through the load resistor. Since current can not flow through the load during the negative half cycles, the output voltage is equal to zero.

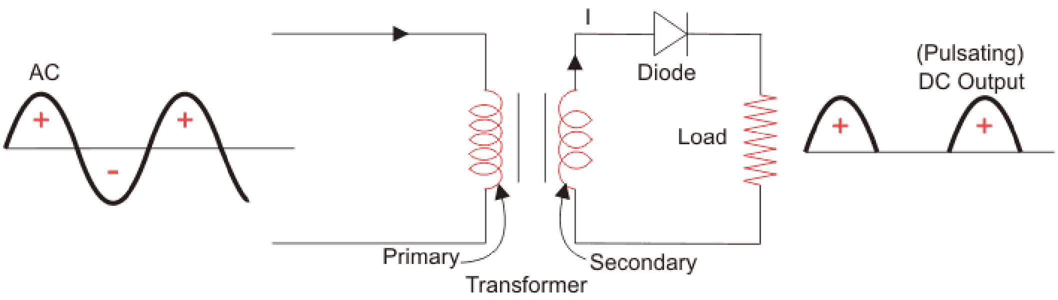

Figure 2: Half wave rectifier circuit diagram and waveform [electrical4u.com].

Therefor, for an AC voltage given by (1) the output voltage of a half wave rectifier will be (for an ideal diode)

$$V_{\mathrm{o}}(t) = \begin{cases} V_m \sin(\omega t), & 0 \leq t \leq T/2 \\ 0, & T/2 \leq t \leq T \end{cases} \tag{8}$$

Average output voltage of a half wave rectifier

To calculate the average voltage, V_{dc}, of the pulsating DC output of a half wave rectifier we use the definition (2). Therefore, for the voltage (8) we have

$$\begin{aligned} V_{\mathrm{dc}} &= \frac{1}{T}\int_0^T V_{\mathrm{o}}(t)\mathrm{d}t \\ &= \frac{1}{T}\int_0^{T/2} V_m \sin(\omega t)\mathrm{d}t + \frac{1}{T}\int_{T/2}^{T} 0\ \mathrm{d}t \\ &= \frac{V_m}{T}\int_0^{T/2} \sin(\omega t)\mathrm{d}t \\ &= \frac{V_m}{T}\left[-\frac{\cos(\omega t)}{\omega}\right]_0^{T/2} \\ &= \frac{V_m}{\omega T}\{-\cos(\omega T/2) + \cos(0)\} \\ &= \frac{V_m}{\pi}. \end{aligned} \tag{9}$$

Here we have used the relation $\omega = 2\pi/T$.

RMS value of the output voltage of a half wave rectifier

To calculate the RMS value of the output voltage, V_{rms}, of the pulsating DC output of a half wave rectifier we use the definition (5). Therefore, for the voltage (8) we have

$$\begin{aligned}
V_{\mathrm{rms}}^2 &= \frac{1}{T}\int_0^T V_{\mathrm{o}}^2(t)\mathrm{d}t \\
&= \frac{V_m^2}{T}\int_0^{T/2} \sin^2(\omega t)\mathrm{d}t + \frac{V_m^2}{T}\int_{T/2}^{T} 0 \,\mathrm{d}t \\
&= \frac{V_m^2}{2T}\int_0^{T/2} 2\sin^2(\omega t)\mathrm{d}t \\
&= \frac{V_m^2}{2T}\int_0^{T/2} \{1-\cos(2\omega t)\}\,\mathrm{d}t \\
&= \frac{V_m^2}{2T}\int_0^{T/2} \mathrm{d}t - \frac{V_m^2}{T}\int_0^{T/2} \cos(2\omega t)\mathrm{d}t \\
&= \frac{V_m^2}{2T}\Big[T\Big]_0^{T/2} - \frac{V_m^2}{2T}\left[\frac{\sin(2\omega t)}{2\omega}\right]_0^{T/2} \\
&= \frac{V_m^2}{4} - \frac{V_m^2}{\omega T}\Big\{\sin(2\omega T) - \sin(0)\Big\} \\
&= \frac{V_m^2}{4}.
\end{aligned} \tag{10}$$

Hence for the half wave rectifier

$$\boxed{V_{\mathrm{rms}} = \frac{V_m}{2}.} \tag{11}$$

Ripple factor of half wave rectifier

Ripple is the unwanted AC component remaining when converting the AC voltage waveform into a DC waveform. Even though we try out best to remove all AC components, there is still some small amount left on the output side which pulsates the DC waveform. This undesirable AC component is called ripple.

To quantify how well the half wave rectifier can convert the AC voltage into DC voltage, we use what is known as the ripple factor (represented by γ). The ripple factor is the ratio between the RMS value of the AC voltage and the DC voltage of the rectifier.

$$\boxed{\gamma = \frac{\text{RMS value of the AC component}}{\text{value of DC component}} = \frac{V_{\mathrm{r(rms)}}}{V_{\mathrm{dc}}}.} \tag{12}$$

Note that the RMS value of the AC component of the signal is $V_{\mathrm{r(rms)}}$ and V_{rms} is the RMS value of the whole voltage signal.

To calculate $V_{\mathrm{r(rms)}}$, the RMS value of the AC component present in the output of the half wave rectifier we write the output voltage as

$$V_{\mathrm{o}}(t) = V_{\mathrm{ac}} + V_{\mathrm{dc}}, \tag{13}$$

where V_{ac} is the AC component remaining when converting the AC voltage waveform into a DC waveform. The RMS value of the AC component present in the output of the half wave rectifier is given by

$$V_{\mathrm{r(rms)}} = \left[\frac{1}{T}\int_0^T V_{\mathrm{ac}}^2 \,\mathrm{d}t\right]^{1/2}. \tag{14}$$

Therefore,

$$\begin{aligned} V_{\mathrm{r(rms)}}^2 &= \frac{1}{T}\int_0^T (V_{\mathrm{o}} - V_{\mathrm{dc}})^2 \,\mathrm{d}t \\ &= \frac{1}{T}\int_0^T (V_{\mathrm{o}}^2 - 2V_{\mathrm{o}}V_{\mathrm{dc}} + V_{\mathrm{dc}}^2) \,\mathrm{d}t \\ &= \frac{1}{T}\int_0^T V_{\mathrm{o}}^2 \,\mathrm{d}t - \frac{2V_{\mathrm{dc}}}{T}\int_0^T V_{\mathrm{o}} \,\mathrm{d}t + V_{\mathrm{dc}}^2 \\ &= V_{\mathrm{rms}}^2 - 2V_{\mathrm{dc}}^2 + V_{\mathrm{dc}}^2 \\ &= V_{\mathrm{rms}}^2 - V_{\mathrm{dc}}^2. \end{aligned} \tag{15}$$

Hence the formula to calculate the ripple factor can be written as

$$\gamma = \frac{V_{\mathrm{r(rms)}}}{V_{\mathrm{dc}}} = \sqrt{\left(\frac{V_{\mathrm{rms}}}{V_{\mathrm{dc}}}\right)^2 - 1} \tag{16}$$

Using the values of V_{dc} and V_{rms} given in (9) and (11) respectively for the half wave rectifier we find the the ripple factor as

$$\gamma = \sqrt{\left(\frac{V_m}{2} \times \frac{\pi}{V_m}\right)^2 - 1} = \sqrt{\left(\frac{\pi}{2}\right)^2 - 1} \approx 1.21. \tag{17}$$

Note that to construct a good rectifier, one should keep the ripple factor as low as possible. This is why capacitors and inductors as filters are used to reduce the ripples in the circuit.

Efficiency of half wave rectifier

The ratio of the DC power available at the load to the applied input AC power is known as the efficiency, η. Mathematically it can be given as:

$$\boxed{\eta = \frac{\text{DC power output}}{\text{AC power input}} = \frac{P_{\mathrm{dc}}}{P_{\mathrm{ac}}}.} \tag{18}$$

Let r_f and R_L be the forward resistance and load resistance of the diode. The voltage appearing across the secondary of the power transformer is given by (1). The waveform diagram at the right side of the Figure 2 shows only a positive waveform at the output and a suppressed negative waveform. During the conduction period the instantaneous value of the current is given by the equation:

$$I(t) = \frac{V(t)}{R_L + r_f} = \frac{V_m}{R_L + r_f} \sin(\omega t) = I_m \sin(\omega t), \tag{19}$$

with $I_m = V_m/(r_f + R_L)$ being the maximum current.

Now, the AC power input to the load is given as,

$$P_{\text{ac}} = I_{\text{rms}}^2 (R_L + r_f) = \frac{V_{\text{rms}}^2}{R_L + r_f}. \tag{20}$$

Since the output is obtained across R_L, the DC power output is given by

$$P_{\text{dc}} = I_{\text{dc}}^2 R_L = \frac{V_{\text{dc}}^2}{R_L}. \tag{21}$$

The half wave rectifier efficiency is then

$$\begin{aligned} \eta &= \frac{P_{\text{dc}}}{P_{\text{ac}}} \\ &= \frac{V_{\text{dc}}^2}{R_L} \times \frac{R_L + r_f}{V_{\text{rms}}^2} \\ &= \frac{V_{\text{dc}}^2}{V_{\text{rms}}^2} \times \frac{R_L + r_f}{R_L} \\ &= \left(\frac{V_{\text{dc}}}{V_{\text{rms}}}\right)^2 \times \left(1 + \frac{r_f}{R_L}\right) \\ &= \left(\frac{V_m/\pi}{V_m/2}\right)^2 \times \left(1 + \frac{r_f}{R_L}\right) \\ &\approx 0.4053 \left(1 + \frac{r_f}{R_L}\right) \end{aligned} \tag{22}$$

In reality r_f is much smaller then R_L. If we neglect r_f compare to R_L then the efficiency of the rectifier is maximum. Therefore,

$$\boxed{\eta_{\text{max}} \approx 0.4053 = 40.53\%.} \tag{23}$$

This indicates that the half wave rectifier can convert maximum 40.53% of AC power into DC power, and the remaining power of 59.47% is lost in the rectifier circuit. In fact, 50% power in the negative half cycle is not converted and the remaining 9.47% is lost in the circuit.

Form factor of half wave rectifier

Form factor (f.f.) is defined as the ratio between RMS load voltage and average load voltage. The form factor of the half wave rectifier is as

$$\text{f.f.} = \frac{V_{\text{rms}}}{V_{\text{dc}}} = \frac{V_m/2}{V_m/\pi} = \frac{\pi}{2} \approx 1.57. \tag{24}$$

The form factor is used to get the information of the waveform.

Peak Inverse Voltage (PIV) of half wave rectifier

Peak Inverse Voltage (PIV) is the maximum voltage that the diode can withstand during reverse bias condition. If a voltage is applied more than the PIV, the diode will be destroyed. The peak-inverse-voltage (PIV) rating of a diode is of the primary importance in the design of rectification systems. During negative half cycles of the input voltage, the diode is reversed biased, no current flows through the load resistance R_L and so causes no voltage drop across load resistance R_L and consequently the whole of the input voltage appears across the diode. Thus the maximum voltage, that appears across the diode, is equal to the peak value of the secondary voltage i.e. V_m. Thus for a half-wave rectifier

$$\text{PIV} = V_m. \tag{25}$$

Peak factor of half wave rectifier

It is defined as the ratio of the peak value of the output voltage to the RMS value of the output voltage. The peak factor of the half wave rectifier is as

$$\text{Peak factor} = \frac{V_m}{V_{\text{rms}}} = 2. \tag{26}$$

Applications of half wave rectifier

Half wave rectifier is not so good as compared to Full-wave or Bridge rectifier, but sometimes we require this rectifier depending on the requirements. Some of the applications of half-wave rectifier are

- It is used for the detection of amplitude modulated radio signals.
- For the welding purpose, it supplies polarized voltage.
- It is used in many signal demodulation processes.

Advantages of half wave rectifier

The main advantage of half-wave rectifiers is in their simplicity. As they do not require as many components, they are simpler and cheaper to setup and construct. As such, the main advantages of half-wave rectifiers are:

- Simple (lower number of components)
- Cheaper up front cost (as their is less equipment. Although there is a higher cost over time due to increased power losses)

Disadvantages of half wave rectifier

The disadvantages of half-wave rectifiers are:

- They only allow a half-cycle through per sinewave, and the other half-cycle is wasted. This leads to power loss.
- They produces a low output voltage.
- The output current we obtain is not purely DC, and it still contains a lot of ripple (i.e. it has a high ripple factor)

3 Full wave rectifier

A full wave rectifier converts both halves of each cycle of an alternating wave (AC signal) into pulsating DC signal. Figure 3 shows the input AC voltage waveform, the circuit diagram and the final output voltage waveform of a center tapped full wave rectifier.

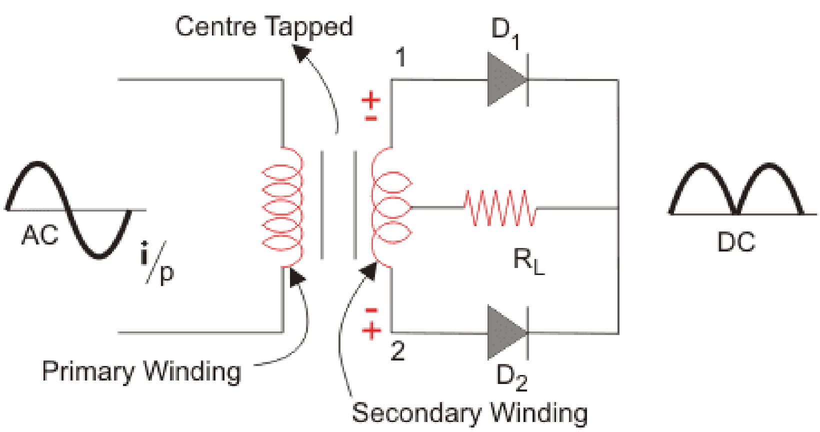

Figure 3: Center tapped full wave rectifier circuit diagram and waveform [electrical4u.com].

For an AC voltage given by (1) the waveform of the output voltage of a full wave rectifier can be written as (for an ideal diode)

$$V_{\rm o}(t) = \begin{cases} V_m \sin(\omega t), & 0 \le t \le T/2 \\ V_m \sin(\omega t - \pi), & T/2 \le t \le T \end{cases} \tag{27}$$

Average output voltage of a full wave rectifier

$$\begin{aligned} V_{\rm dc} &= \frac{1}{T}\int_0^T V_{\rm o}(t){\rm d}t \\ &= \frac{1}{T/2}\int_0^{T/2} V_m \sin(\omega t){\rm d}t \\ &= \frac{2V_m}{T}\int_0^{T/2} \sin(\omega t){\rm d}t \\ &= \frac{2V_m}{\pi}. \end{aligned} \tag{28}$$

RMS value of the output voltage of a full wave rectifier

$$\begin{aligned} V_{\rm rms} &= \left[\frac{1}{T}\int_0^T V_{\rm o}^2(t){\rm d}t\right]^{1/2} \\ &= \left[\frac{V_m^2}{T/2}\int_0^{T/2} \sin^2(\omega t){\rm d}t\right]^{1/2} \\ &= \left[\frac{V_m^2}{T}\int_0^{T/2} 2\sin^2(\omega t){\rm d}t\right]^{1/2} \\ &= \frac{V_m}{\sqrt{2}}. \end{aligned} \tag{29}$$

Ripple factor of full wave rectifier

$$\begin{aligned} \gamma &= \sqrt{\left(\frac{V_{\rm rms}}{V_{\rm dc}}\right)^2 - 1} \\ &= \sqrt{\left(\frac{\pi}{2\sqrt{2}}\right)^2 - 1} \\ &\approx 0.48 \end{aligned} \tag{30}$$

Efficiency of full wave rectifier

$$\begin{aligned}\eta &= \frac{P_{\text{dc}}}{P_{\text{dc}}} \\ &= \left(\frac{V_{\text{dc}}}{V_{\text{rms}}}\right)^2 \times \left(1+\frac{r_f}{R_L}\right) \\ &\approx 0.8106\left(1+\frac{r_f}{R_L}\right)\end{aligned} \tag{31}$$

In reality r_f is much smaller then R_L. If we neglect r_f compare to R_L then the efficiency of the rectifier is maximum. Therefore,

$$\boxed{\eta_{\text{max}} \approx 0.8106 = 81.06\%.} \tag{32}$$

Form factor of full wave rectifier

$$\text{Form factor} = \frac{V_{\text{rms}}}{V_{\text{dc}}} = \frac{\pi}{2\sqrt{2}} \approx 1.11. \tag{33}$$

Peak Inverse Voltage (PIV) of full wave rectifier

Peak inverse voltage(PIV) or peak reverse voltage(PRV) can be defined as the maximum value of the reverse voltage of a diode, which occurs at the peak of the input cycle when the diode is in reverse bias.

PIV of center tapped full wave rectifier is $2V_m$ and of a bridge rectifiers it is V_m.

Peak factor of full wave rectifier

$$\text{Peak factor} = \frac{V_m}{V_{\text{rms}}} = \sqrt{2}. \tag{34}$$

Applications of full wave rectifier

Full wave rectifier is of two types; center tapped and bridge rectifier. Both these rectifiers are used for following purposes depends upon the requirement. Following of full wave rectifier applications are:

- It can be used to detect the amplitude of modulated radio signal.

- It can be used to supply polarized voltage in welding.
- The Bridge Rectifier circuits are widely used in power supply for various appliances, as they are capable of converting the High AC voltage into Low DC voltage.

Advantages of full wave rectifier

- Full wave rectifiers have higher rectifying efficiency than half-wave rectifiers. This means that they convert AC to DC more efficiently.
- They have low power loss because no voltage signal is wasted in the rectification process.
- The output voltage of center tapped full wave rectifier has lower ripples than a half wave rectifiers.

Disadvantages of full wave rectifier

- The center tapped rectifier is more expensive than half-wave rectifier and tends to occupy a lot of space.

A comparison of different parameters related to the half and full wave rectifiers are given below:

Parameters	Half wave rectifier	Full wave rectifier
Number of diodes	1	2 or 4
Maximum efficiency	40.53%	81.06 %
Peak inverse voltage	V_m	V_m or $2V_m$
Average voltage no load	V_m/π	$2V_m/\pi$
V_{rms} no load	$V_m/2$	$V_m/\sqrt{2}$
Ripple factor	1.21	0.48
Form factor	1.57	1.11
Output frequency	f	$2f$

4 RC circuit

An *RC* circuit (also known as an *RC* filter or *RC* network) stands for a resistor-capacitor circuit. An *RC* circuit is defined as an electrical circuit composed of the passive circuit components of a resistor (R) and capacitor (C), driven by a voltage source or current source. In an RC series circuit, a pure resistor having resistance R in ohms and a pure capacitor of capacitance C in Farads are connected in series.

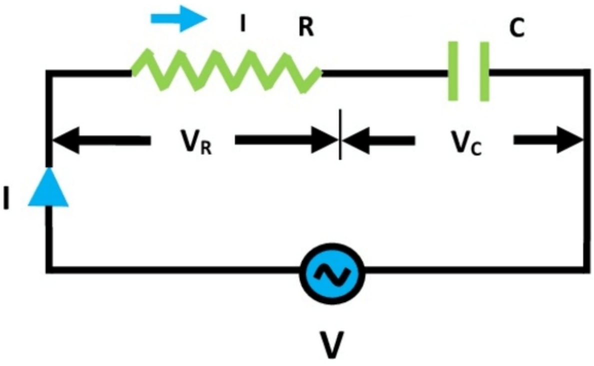

Figure 4: RC circuit diagram [electrical4u.com].

RC time constant

The RC time constant indicates the rate of charge or discharge. RC specifies the time it takes C to charge to 63% of the charging voltage. Similarly, RC specifies the time it takes C to discharge 63% of the way down to the value equal to 37% of the initial voltage across C at the start of discharge.

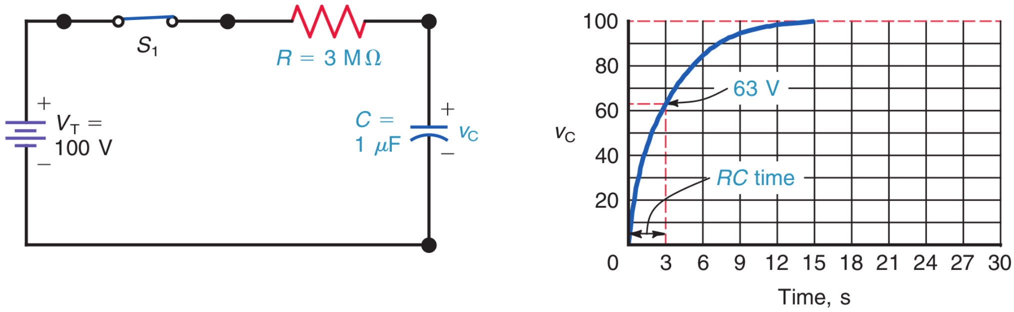

Figure 5: C charges through R to 63% of V_T in one RC time constant.

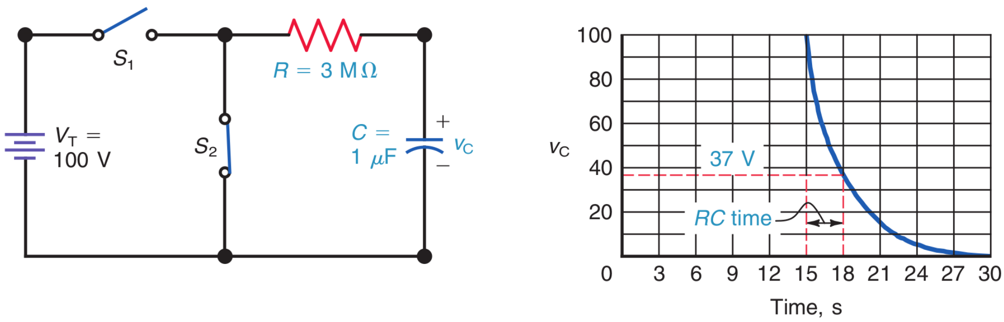

Figure 6: C to discharge through R, V_C drops to 37% of its initial voltage in one time constant.

5 Half wave rectifier with capacitor filter

Filters are components used to convert (smoothen) pulsating DC waveforms into constant DC waveforms. They achieve this by suppressing the DC ripples in the waveform.

Although half-wave rectifiers without filters are theoretically possible, they can not be used for any practical applications. As DC equipment requires a constant waveform, we need to smooth out this pulsating waveform for it to be any use in the real world. This is why in reality we use half wave rectifiers with a filter. A capacitor or an inductor can be used as a filter – but half wave rectifier with capacitor filter is most commonly used. The circuit diagram below shows how a capacitive filter is can be used to smoothen out a pulsating DC waveform into a constant DC waveform.

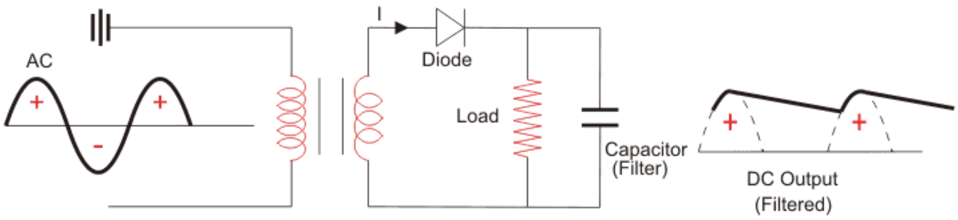

Figure 7: Half wave rectifier with capacitor filter and waveform [electrical4u.com].

6 Full wave rectifier with capacitor filter

With the center tapped full wave rectifier we get a pulsating DC voltage with a lot of ripples as the output. We cannot use this pulsating for practical applications. So, to convert the pulsating DC voltage to pure DC voltage, we use a filter circuit as shown above. Here we place a capacitor across the load. The working of the capacitive filter

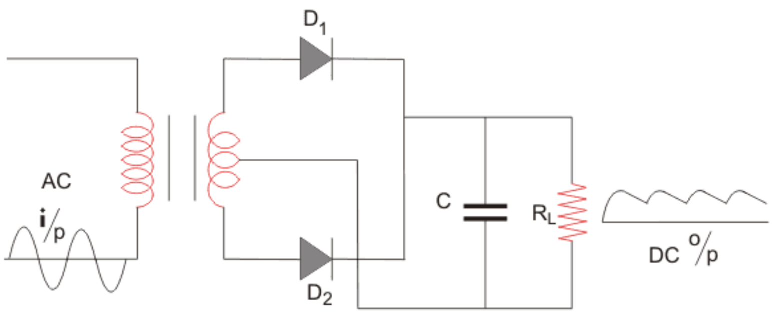

Figure 8: Full wave rectifier circuit diagram with capacitor filter [electrical4u.com].

circuit is to short the ripples and block the DC component so that it flows through another path and is available across the load. During the positive half-wave, the diode D_1 starts conducting. The capacitor is uncharged, and when we apply an input AC voltage which happens to be more than the capacitor voltage, it charges the capacitor immediately to the maximum value of the input voltage. At this point, the supply voltage is equal to capacitor voltage.

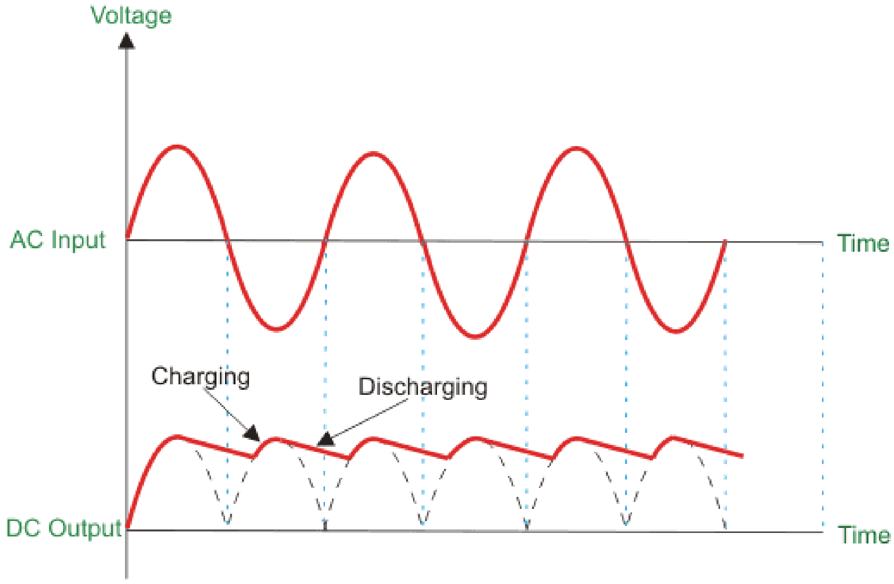

Figure 9: Waveform of full wave rectifier *RC* filter [electrical4u.com].

When the applied AC voltage starts decreasing and less than the capacitor, the capacitor starts discharging slowly but this is slower when compared to the charging of the capacitor and it does not get enough time to discharge entirely and the charging starts again. So around half of the charge present in the capacitor gets discharged. During the negative cycle, the diode D_2 starts conducting, and the above process happens again. This will cause the current to flow through the same direction across the load.