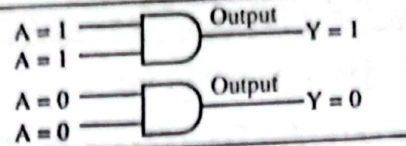
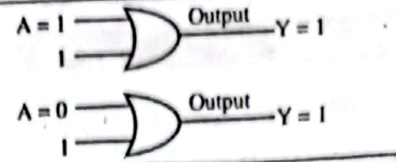
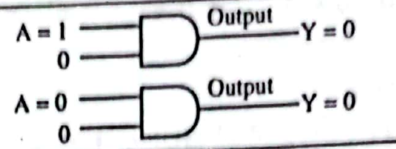
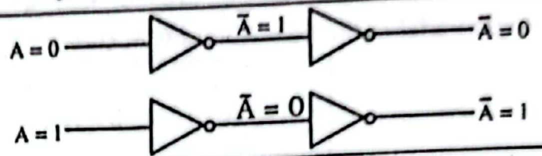
| | |
|---|---|
| Theorem 6 . (A. A = A) |   |
| Theorem 7 . (A + 1 = 1) |   |
| Theorem 8 . (A. 0 = 0) |   |
| Theorem 9 . ($\bar{\bar{A}}$ = A) |   |

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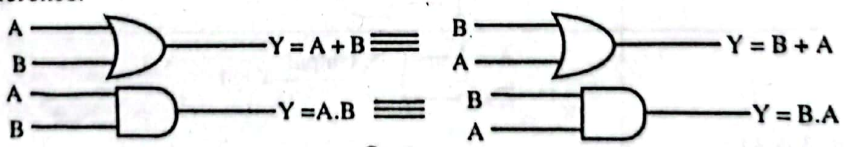
**❑ মাল্টিভেরিয়েবল থিওরেম (Multivariable theorem) ঃ**

লজিক গেইটের ইনপুটের একের অধিক ভেরিয়েবল (Variable) থাকলে এবং তার লজিক ফাংশনকে বুলিয়ান থিওরেম মাধ্যমে প্রকাশ করা হলে, তাকে Multivariable theorem বলে।

নিচের টেবিলে Multivariable theorem দেয়া হলো–

| | | |
|---|---|---|
| Theorem 10 : | A + B = B + A | Commutative Law |
| Theorem 11 : | A . B = B. A | |
| Theorem 12 : | A + (B + C) = (A + B) + C | Associative Law |
| Theorem 13 : | A . (B . C) = (A . B) . C | |
| Theorem 14 : | A . (B + C) = A . B + A . C | Distributive Law |
| Theorem 15 : | (A + B ) . (C + D) = A . C + B . C + A. D + B. D | |
| Theorem 16 : | A + A . B = A | |
| Theorem 17 : | $(\overline{A + B}) = \bar{A} . \bar{B}$ | De Morgan's Theorems |
| Theorem 18 : | $(\overline{A . B}) = \bar{A} + \bar{B}$ | |

(a) Theorems 10 and 11 obey commutative law. This law states that the order in which the variables are ORed or AND makes no difference.

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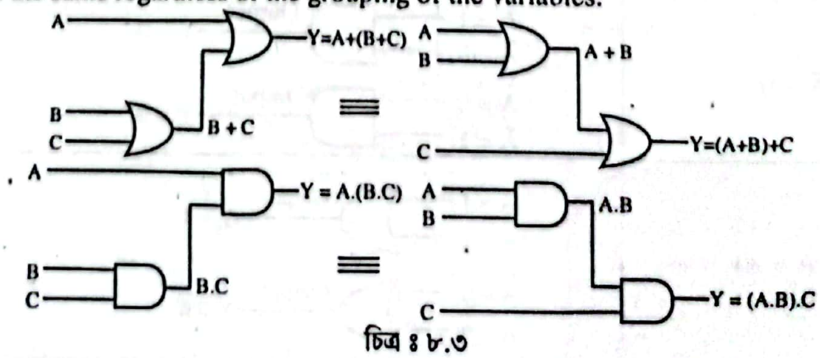
(b) Theorems 12 and 13 obey associative law. This law states that in the ORing or ANDing of several variables, the result is the same regardless of the grouping of the variables.

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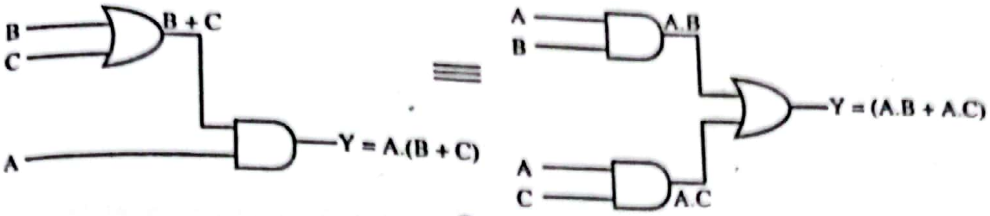
(c) Theorems 14 and 15 obey distributive law. This law states that a Boolean expression can be expanded by multiplying term-by-term just the same as in ordinary algebra.

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(d) We will prove Theorem 16 by factoring and using Theorems 2, 7, 10 and 14.

$A + A.B = A.1 + A.B$ ................. Theorem 2

$= A.(1 + B)$ ................. Theorem 14

$= A.(B + 1)$ ................. Theorem 10

$= A.1$ ................. Theorem 7

$= A$ ................. Theorem 2

(e) Theorems 17 and 18 are the two most important theorems of Boolean algebra and were contributed by the great mathematician named De Morgan. Therefore, these theorems are called De Morgan's theorems.

**উদাহরণ-১ (ক) ঃ Using Boolean algebraic techniques, simplify the following expression :**

$$Y = A.B.\bar{C}.\bar{D} + \bar{A}.B.\bar{C}.\bar{D} + \bar{A}.B.C.\bar{D} + A.B.C.\bar{D}$$

**Solution :** $Y = A.B.\bar{C}.\bar{D} + \bar{A}.B.\bar{C}.\bar{D} + \bar{A}.B.C.\bar{D} + A.B.C\bar{D}$ .. (i)

Step 1 : Take out the common factors as below :

$$Y = B\bar{C}\bar{D}(A + \bar{A}) + BC\bar{D}(A + \bar{A})$$

Step 2 : Apply Theorem 3 $(A + \bar{A} = 1)$ :

$$Y = B\bar{C}\bar{D} + BC\bar{D}$$

Step 3 : Again factorize :

$$Y = B\bar{D}(C + \bar{C})$$

Step 4 : Apply Theorem 3 $(C + \bar{C} = 1)$ :

$$Y = B\bar{D}.1 = B\bar{D}$$

This is the simplified form of exp. 1.

**(খ) Using Boolean techniques, simplify the following expression :**

$$Y = AB + A(B + C) + B(B + C)$$

**Solution :** $Y = AB + A(B + C) + B(B + C)$... (i)

Step 1 : Apply Theorem 14 (distributive law) to second and third terms :

$$Y = AB + AB + AC + BB + BC$$

Step 2 : Apply Theorem 6 $(B.B = B)$ :

$$Y = AB + AB + AC + B + BC$$

Step 3 : Apply Theorem 5 $(AB + AB = AB)$ :

$$Y = AB + AC + B + BC$$

Step 4 : Factor B out of last 2 terms :

$$Y = AB + AC + B(1 + C)$$

Step 5 : Apply commutative law and Theorem 7 $(1 + C = C + 1 = 1)$ :

$$Y = AB + AC + B.1$$

Step 6 : Apply Theorem 2 $(B.1 = B)$ :

$$Y = AB + AC + B$$

Step 7 : Factor B out of first and third terms :

$$Y = B(A + 1) + AC$$

Step 8 : Apply Theorem 7 $(A + 1 = 1)$ :

$$Y = B.1 + AC$$

Step 9 : Apply Theorem 2 $(B.1 = B)$ :

$$Y = B + AC$$

This is the simplified form of exp. 2.

**উদাহরণ-২ (ক) ঃ Simplify the following Boolean expressions to a minimum number of literals ;**

(i) $Y = A + \bar{A}B$ (ii) $Y = AB + \bar{A}C + BC$

**Solution. (i) :**

$$Y = A + \bar{A}B$$

$$= A + AB + \bar{A}B \quad [\because A = A + AB \text{ from Theorem 16}]$$

$$= A + B(A + \bar{A})$$

$$= A + B \quad [\because A + \bar{A} = 1 \text{ from Theorem 3}]$$

$$\therefore Y = A + B$$

**(ii)**

$$Y = AB + \bar{A}C + BC$$

$$= AB + \bar{A}C + BC.(A + \bar{A})$$

$$= AB + \bar{A}C + ABC + \bar{A}BC$$

$$= AB + ABC + \bar{A}C + \bar{A}BC$$

$$= AB(1 + C) + \bar{A}C(1 + B)$$

$$= AB + \bar{A}C \quad [\because A + 1 = 1 \text{ from theorem 7}]$$

$$Y = AB + \bar{A}C$$

(খ) Simplify the following Boolean expression :

$$Y = (\bar{A} + B)(A + B)$$

**Solution :**

$$Y = (\bar{A} + B)(A + B)$$

The expression can be expanded by multiplying out the terms (Theorem 15].

$$Y = \bar{A}.A + \bar{A}.B + B.A + B.B$$

Using Theorem 4, $\bar{A}.A = 0$, Also $B.B = B$ [Theorem 6].

$$\therefore Y = 0 + \bar{A}.B + B.A + B$$

$$= \bar{A}.B + AB + B$$

Factoring out the variable B [Theorem 14], we have,

$$Y = B(\bar{A} + A + 1)$$

Using theorem 7, $A + 1 = 1$

$$\therefore Y = B(\bar{A} + 1)$$

Again using Theorem 7, $\bar{A} + 1 = 1$.

$$\therefore Y = B.1$$

Finally, using Theorem 2, we have,

$$Y = B$$