

8) । যদি $x = \frac{1}{2} (-1 + \sqrt{-3})$ ও $y = \frac{1}{2} (-1 - \sqrt{-3})$ হয়, তবে প্রমাণ কর যে,

$$\text{i) } x^3 + y^{-3} = 2$$

$$\text{ii) } x^4 + x^2 y^2 + y^4 = 0$$

$$\begin{aligned} x &= \frac{1}{2} (-1 + \sqrt{-3}) \\ &= \omega \end{aligned}$$

$$\begin{aligned} y &= \frac{1}{2} (-1 - \sqrt{-3}) \\ &= \omega^2 \end{aligned}$$

i) L. H. S.

$$= x^3 + y^{-3}$$

$$= x^3 + \frac{1}{y^3}$$

$$= \omega^3 + \frac{1}{(\omega^2)^3}$$

$$= \omega^3 + \frac{1}{\omega^6}$$

$$\begin{aligned}
&= \omega^3 + \frac{1}{(\omega^3)^2} \\
&= 1 + \frac{1}{1} \\
&= 2 = R.H.S.
\end{aligned}$$

ii) $L.H.S.$

$$\begin{aligned}
&= x^4 + x^2y^2 + y^4 \\
&= \omega^4 + \omega^2(\omega^2)^2 + (\omega^2)^4 \\
&= \omega^3\omega + \omega^6 + \omega^8 \\
&= \omega + (\omega^3)^2 + \omega^6\omega^2 \\
&= \omega + 1 + (\omega^3)^2\omega^2 \\
&= \omega + 1 + \omega^2 \\
&= 0 = R.H.S.
\end{aligned}$$

9) এককের একটি কাল্পনিক ঘনমূল ω হলে, প্রমাণ কর যে,

$$\text{i) } (1 - \omega^2)(1 - \omega^4)(1 - \omega^8)(1 - \omega^{10}) = 9$$

$$\text{ii) } (1 - \omega + \omega^2)(1 - \omega^2 + \omega^4)(1 - \omega^4 + \omega^8)(1 - \omega^8 + \omega^{16}) = 16$$

$$\begin{aligned} \text{i) L. H. S.} &= (1 - \omega^2)(1 - \omega^4)(1 - \omega^8)(1 - \omega^{10}) \\ &= (1 - \omega^2)(1 - \omega^3\omega)(1 - \omega^6\omega^2)(1 - \omega^9\omega) \\ &= (1 - \omega^2)(1 - \omega)\{1 - (\omega^3)^2\omega^2\}\{1 - (\omega^3)^3\omega\} \\ &= (1 - \omega^2)(1 - \omega)(1 - \omega^2)(1 - \omega) \\ &= (1 - \omega^2)^2(1 - \omega)^2 \\ &= (1 - 2\omega^2 + \omega^4)(1 - 2\omega + \omega^2) \\ &= (1 - 2\omega^2 + \omega)(1 - 2\omega + \omega^2) \\ &= (1 + \omega^2 - 3\omega^2 + \omega)(1 + \omega - 3\omega + \omega^2) \\ &= (1 + \omega^2 - 3\omega^2 + \omega)(1 + \omega - 3\omega + \omega^2) \\ &= (0 - 3\omega^2)(0 - 3\omega) \\ &= 9\omega^3 = 9 \end{aligned}$$