

$$i) \quad {}^nC_r = {}^nC_{n-r}$$

$$\text{আমরা জানি, } {}^nC_r = \frac{n!}{r! \times (n-r)!} \dots \dots \dots (i)$$

(i)নং সমীকরণে  $r = n - r$  বসিয়ে পাই,

$$\begin{aligned} {}^nC_{n-r} &= \frac{n!}{(n-r)! \times (n-n+r)!} \\ &= \frac{n!}{(n-r)! \times (n-n+r)!} \\ &= \frac{n!}{r! \times (n-r)!} \\ &= {}^nC_r \end{aligned}$$

$$\therefore {}^nC_r = {}^nC_{n-r}$$

$$\text{ii)} \quad {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$$

L.H.S.

$$\begin{aligned}
 & {}^nC_r + {}^nC_{r-1} \\
 &= \frac{n!}{r! \times (n-r)!} + \frac{n!}{(r-1)! \times (n-r+1)!} \\
 &= \frac{n!}{r(r-1)! \times (n-r)!} + \frac{n!}{(r-1)! \times (n-r+1)(n-r+1-1)!} \\
 &= \frac{n!}{r(r-1)! \times (n-r)!} + \frac{n!}{(r-1)! \times (n-r+1)(n-r)!} \\
 &= \frac{n!}{(r-1)! \times (n-r)!} \left[ \frac{1}{r} + \frac{1}{n-r+1} \right] \\
 &= \frac{n!}{(r-1)! \times (n-r)!} \left[ \frac{n-r+1+r}{r(n-r+1)} \right] \\
 &= \frac{n!(n+1)}{r(r-1)! \times (n-r)!(n-r+1)} \\
 &= \frac{(n+1)!}{r! \times (n-r+1)!} \\
 &= {}^{n+1}C_r \\
 &= \text{R.H.S.}
 \end{aligned}$$

13)  ${}^nC_{10} = {}^nC_{20}$  হলে  $n$  এর মান কত?

14)  ${}^nC_8 = {}^nC_{12}$  হলে  $n$  এর মান কত?