সূচক ধারা

EXPONENTIAL SERIES

CH 4

1) $e^x = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} \ldots\ldots\ldots\ldots\ldots$

2) $e^{-x} = 1 - \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} - \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} - \ldots\ldots\ldots\ldots\ldots$

3) $\frac{e^x + e^{-x}}{2} = (1 + \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} + \frac{x^6}{\lfloor 6} + \ldots\ldots\ldots)$

4) $\frac{e^x - e^{-x}}{2} = (\frac{x}{\lfloor 1} + \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} + \frac{x^7}{\lfloor 7} + \ldots\ldots\ldots)$

5) $e = 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \ldots\ldots\ldots\ldots\infty$

6) $e^{-1} = 1 - \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} - \frac{1}{\lfloor 3} + \ldots\ldots\ldots\ldots\infty$

7) $\frac{e^1 + e^{-1}}{2} = \left(1 + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 4} + \frac{1}{\lfloor 6} + \ldots\ldots\ldots\right)$

8) $\frac{e^1 - e^{-1}}{2} = \left(1 + \frac{1}{\lfloor 3} + \frac{1}{\lfloor 5} + \frac{1}{\lfloor 7} + \ldots\ldots\ldots\right)$

9) $a^x = 1 + \frac{(x\log a)}{\lfloor 1} + \frac{(x\log a)^2}{\lfloor 2} + \frac{(x\log a)^3}{\lfloor 3} + \ldots\ldots\ldots$

10) $a^{-x} = 1 - \frac{(x\log a)}{\lfloor 1} + \frac{(x\log a)^2}{\lfloor 2} - \frac{(x\log a)^3}{\lfloor 3} + \ldots\ldots\ldots$

Page # 116

5) দেখাও যে, $\frac{2}{\lfloor 1} + \frac{6}{\lfloor 2} + \frac{12}{\lfloor 3} + \frac{20}{\lfloor 4} + \ldots\ldots\ldots\ldots = 3e$

$$L.H.S = \frac{2}{\lfloor 1} + \frac{6}{\lfloor 2} + \frac{12}{\lfloor 3} + \frac{20}{\lfloor 4} + \ldots\ldots\ldots\ldots\ldots\ldots..$$

$$= \frac{1.2}{\lfloor 1} + \frac{2.3}{\lfloor 2} + \frac{3.4}{\lfloor 3} + \frac{4.5}{\lfloor 4} + \ldots\ldots\ldots\ldots\ldots\ldots..$$

ধারাটির n - তম পদ $= \frac{\{1+(n-1)1\}\{2+(n-1)1\}}{\lfloor n}$

$$= \frac{(1+n-1)(2+n-1)}{\lfloor n}$$

$$= \frac{n(n+1)}{\lfloor n} \quad = \frac{n(n+1)}{n \lfloor n-1} \qquad = \frac{(n+1)}{\lfloor n-1}$$

$$= \frac{n-1+2}{\lfloor n-1}$$

$$= \frac{n-1}{\lfloor n-1} + \frac{2}{\lfloor n-1}$$

$$= \frac{n-1}{(n-1)\lfloor n-2} + \frac{2}{\lfloor n-1}$$

$$= \frac{1}{\lfloor n-2} + \frac{2}{\lfloor n-1}$$

n এর পরিবর্তে 1,2,3,..... .....ইতাদি বসাইয়া পাই,

$$t_1 = \frac{1}{\lfloor 1-2} + \frac{2}{\lfloor 1-1} = \frac{1}{\lfloor -1} + \frac{2}{\lfloor 0}$$

$$t_2 = \frac{1}{\lfloor 2-2} + \frac{2}{\lfloor 2-1} = \frac{1}{\lfloor 0} + \frac{2}{\lfloor 1}$$

$$t_3 = \frac{1}{\lfloor 3-2} + \frac{2}{\lfloor 3-1} = \frac{1}{\lfloor 1} + \frac{2}{\lfloor 2}$$

$$t_4 = \frac{1}{\lfloor 4-2} + \frac{2}{\lfloor 4-1} = \frac{1}{\lfloor 2} + \frac{2}{\lfloor 3}$$

.....

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এই পদগুলো যোগ করে পাই,

S=$t_1 + t_2 + t_3 + t_4 + \ldots\ldots\ldots\ldots$

$= \left(\frac{1}{\lfloor -1} + \frac{1}{\lfloor 0} + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \ldots..\right) + \left(\frac{2}{\lfloor 0} + \frac{2}{\lfloor 1} + \frac{2}{\lfloor 2} + \frac{2}{\lfloor 3} + \ldots....\right)$

$= \left(\frac{1}{\infty} + \frac{1}{1} + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \ldots\ldots\right) + \left(\frac{2}{1} + \frac{2}{\lfloor 1} + \frac{2}{\lfloor 2} + \frac{2}{\lfloor 3} + \ldots.\right)$

$= \left(0 + 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \ldots\ldots\right) + 2\left(1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \ldots.\right)$

$= \left(1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \ldots\ldots\right) + 2\left(1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \ldots.\right)$

= e +2e = 3e proved .