সূচক ধারা

EXPONENTIAL SERIES

CH 4

Definition of e:

The limiting value of $(1 + \frac{1}{n})^n$ which n tends to infinity is denoted by e

$$e = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n = 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \ldots\ldots\ldots.\ldots\infty$$

$$(1+x)^n = 1 + \frac{nx}{\underline{|1}} + \frac{n(n-1)}{\underline{|2}} x^2 + \frac{n(n-1)(n-2)}{\underline{|3}} x^3 + \ldots\ldots\ldots.\ldots$$

x এর পরিবর্তে $\frac{1}{n}$ বসাইয়া পাই,

$$(1+\frac{1}{n})^n = 1 + \frac{n\frac{1}{n}}{\underline{|1}} + \frac{n(n-1)}{\underline{|2}}\left(\frac{1}{n}\right)^2 + \frac{n(n-1)(n-2)}{\underline{|3}}\left(\frac{1}{n}\right)^3 + \ldots\ldots\ldots\ldots.\ldots\infty$$

$$= 1 + \frac{1}{\underline{|1}} + \frac{n\,n\,(1-\frac{1}{n})}{\underline{|2}}\ \frac{1}{n^2} + \frac{n\,n\left(1-\frac{1}{n}\right)n\,(1-\frac{2}{n})}{\underline{|3}}\ \frac{1}{n^3} + \ldots\ldots\ldots\ldots\ldots\infty$$

$$= 1 + \frac{1}{\underline{|1}} + \frac{n^2(1-\frac{1}{n})}{\underline{|2}}\ \frac{1}{n^2} + \frac{n^3\left(1-\frac{1}{n}\right)(1-\frac{2}{n})}{\underline{|3}}\ \frac{1}{n^3} + \ldots\ldots\ldots\ldots\ldots.\ldots\infty$$

$$= 1 + \frac{1}{\underline{|1}} + \frac{(1-\frac{1}{n})}{\underline{|2}} + \frac{\left(1-\frac{1}{n}\right)(1-\frac{2}{n})}{\underline{|3}} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\infty$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 1 + \frac{1}{\underline{|1}} + \frac{(1-\frac{1}{\infty})}{\underline{|2}} + \frac{\left(1-\frac{1}{\infty}\right)(1-\frac{2}{\infty})}{\underline{|3}} + \ldots\ldots\ldots\ldots\ldots\ldots\infty$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 1 + \frac{1}{1!} + \frac{(1-0)}{2!} + \frac{(1-0)\,(1-0)}{3!} + \ldots\ldots\ldots\ldots\infty$$

$$= 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots\ldots\ldots\ldots\infty$$

$= 2.7182818$

$= e$

1) $e^{x} = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} \ldots \ldots \ldots \ldots \ldots.$

x এর পরিবর্তে - x বসাইয়া পাই,

2) $e^{-x} = 1 - \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} - \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} - \ldots \ldots \ldots \ldots \ldots.$

(1) + (2) করে পাই,

$$e^{x} = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} \ldots\ldots\ldots\ldots\ldots$$

$$e^{-x} = 1 - \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} - \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} - \ldots\ldots\ldots\ldots\ldots$$

$$e^{x} + e^{-x} = 2\left(1 + \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} + \frac{x^6}{\lfloor 6} + \ldots\ldots\ldots\right)$$

3) $$\frac{e^{x} + e^{-x}}{2} = \left(1 + \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} + \frac{x^6}{\lfloor 6} + \ldots\ldots\ldots\right)$$

(1) - (2) করে পাই,

$$e^x = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} \ldots\ldots\ldots\ldots\ldots.$$

$$e^{-x} = 1 - \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} - \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} - \ldots\ldots\ldots\ldots\ldots.$$

$$e^x - e^{-x} = 2\left(\frac{x}{\lfloor 1} + \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} + \frac{x^7}{\lfloor 7} + \ldots\ldots\ldots\right)$$

4) $$\frac{e^x - e^{-x}}{2} = \left(\frac{x}{\lfloor 1} + \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} + \frac{x^7}{\lfloor 7} + \ldots\ldots\ldots\right)$$

5) $e^x = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} \ldots \ldots \ldots \ldots \ldots.$

x এর পরিবর্তে 1 বসাইয়া পাই,

$$e = 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \ldots \ldots \ldots . \ldots \infty$$

6) $e^x = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} \ldots \ldots \ldots \ldots \ldots.$

x এর পরিবর্তে - 1 বসাইয়া পাই,

$$e^{-1} = 1 - \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} - \frac{1}{\lfloor 3} + \ldots \ldots \ldots . \ldots \infty$$

(5) + (6) করে পাই,

$$e = 1 + \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} + \frac{1}{\underline{|3}} + \ldots\ldots\ldots.\ldots\infty$$

$$e^{-1} = 1 - \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} - \frac{1}{\underline{|3}} + \ldots\ldots\ldots.\ldots\infty$$

$$e^{1} + e^{-1} = 2\left(1 + \frac{1}{\underline{|2}} + \frac{1}{\underline{|4}} + \frac{1}{\underline{|6}} + \ldots\ldots\ldots\right)$$

7) $$\frac{e^{1} + e^{-1}}{2} = \left(1 + \frac{1}{\underline{|2}} + \frac{1}{\underline{|4}} + \frac{1}{\underline{|6}} + \ldots\ldots\ldots\right)$$

(5) - (6) করে পাই,

$$e = 1 + \frac{1}{\underline{\lfloor 1}} + \frac{1}{\underline{\lfloor 2}} + \frac{1}{\underline{\lfloor 3}} + \ldots\ldots\ldots.\ldots\infty$$

$$e^{-1} = 1 - \frac{1}{\underline{\lfloor 1}} + \frac{1}{\underline{\lfloor 2}} - \frac{1}{\underline{\lfloor 3}} + \ldots\ldots\ldots.\ldots\infty$$

$$e^{1} - e^{-1} = 2\left(\frac{1}{\underline{\lfloor 1}} + \frac{1}{\underline{\lfloor 3}} + \frac{1}{\underline{\lfloor 5}} + \frac{1}{\underline{\lfloor 7}} + \ldots\ldots\ldots\right)$$

8) $$\frac{e^{1} - e^{-1}}{2} = \left(1 + \frac{1}{\underline{\lfloor 3}} + \frac{1}{\underline{\lfloor 5}} + \frac{1}{\underline{\lfloor 7}} + \ldots\ldots\ldots\right)$$

$$e^x = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} \dots\dots\dots\dots\dots.$$

x এর পরিবর্তে $\text{xlog}_e a$ বসাইয়া পাই,

$$e^{\text{xlog}_e a} = 1 + \frac{(\text{xlog } a)}{\lfloor 1} + \frac{(x \log a)^2}{\lfloor 2} + \frac{(x \log a)^3}{\lfloor 3} + \dots\dots\dots\dots$$

9) $$a^x = 1 + \frac{(\text{xlog } a)}{\lfloor 1} + \frac{(x \log a)^2}{\lfloor 2} + \frac{(x \log a)^3}{\lfloor 3} + \dots\dots\dots$$

10) $$a^{-x} = 1 - \frac{(\text{xlog } a)}{\lfloor 1} + \frac{(x \log a)^2}{\lfloor 2} - \frac{(x \log a)^3}{\lfloor 3} + \dots\dots\dots.$$

$$\mathbf{e}^{\text{xlog}_e a}$$

$$= e^{\log_e a^x}$$

$$= a^x$$

$$\text{mlog}\, x = \log x^m$$

$$\text{xlog}_e a$$

$$= \log_e a^x$$

$$e^{\log_e x} = x$$

$$3^{\log_3 x} = x$$