সূচক ধারা

EXPONENTIAL SERIES

CH 4

9) প্রমাণ কর যে, $y + \frac{1}{y} = 2\left\{1 + \frac{(\log_e y)^2}{\lfloor 2} + \frac{(\log_e y)^4}{\lfloor 4} + \ldots\ldots\right\}$

সমাধানঃ

$$e^x = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} \ldots\ldots\ldots\ldots\ldots (i)$$

$$e^{-x} = 1 - \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} - \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} - \ldots\ldots\ldots\ldots\ldots (ii)$$

(i) ও (ii) যোগ করে পাই,

$$e^x + e^{-x} = \left(2 + \frac{2x^2}{\lfloor 2} + \frac{2x^4}{\lfloor 4} + \frac{2x^6}{\lfloor 6} + \ldots\ldots\right)$$

$$= 2\left(1 + \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} + \frac{x^6}{\lfloor 6} + \ldots\ldots\right) \quad -----(iii)$$

$e^x = y$ ধরলে পাই,

$$\Rightarrow \frac{1}{e^x} = \frac{1}{y}$$

$$\Rightarrow e^{-x} = \frac{1}{y}$$

$$e^x = y$$

$$\Rightarrow \log_e e^x = \log_e y$$

$$\Rightarrow x = \log_e y$$

(iii) নং সমীকরনে মান বসাইয়া পাই,

$$e^x + e^{-x} = 2\left(1 + \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} + \frac{x^6}{\lfloor 6} + \dots\dots\dots\right)$$

$$\Rightarrow y + \frac{1}{y} = 2\left\{1 + \frac{(\log_e y)^2}{\lfloor 2} + \frac{(\log_e y)^4}{\lfloor 4} + \dots\dots\right\}$$

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8(ii)

দেখাও যে, $\frac{1^2}{\underline{\lfloor 1}} + \frac{2^2}{\underline{\lfloor 2}} + \frac{3^2}{\underline{\lfloor 3}} + \frac{4^2}{\underline{\lfloor 4}} \ldots.\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

ধারাটির n - তম পদ $= \frac{n^2}{\underline{\lfloor n}} = \frac{n^2}{n\underline{\lfloor n-1}}$

$$= \frac{n}{\underline{\lfloor n-1}} \quad = \frac{n-1+1}{\underline{\lfloor n-1}}$$

$$= \frac{n-1}{\underline{\lfloor n-1}} + \frac{1}{\underline{\lfloor n-1}}$$

$$= \frac{n-1}{(n-1)\ \underline{\lfloor n-2}} + \frac{1}{\underline{\lfloor n-1}}$$

$$= \frac{1}{\underline{\lfloor n-2}} + \frac{1}{\underline{\lfloor n-1}}$$

ধারাটির n - তম পদ $= \frac{1}{\lfloor\underline{n-2}} + \frac{1}{\lfloor\underline{n-1}}$

n এর পরিবর্তে 1,2,3,....... ......ইতাদি বসাইয়া পাই,

$$t_1 = \frac{1}{\lfloor\underline{1-2}} + \frac{1}{\lfloor\underline{1-1}} = \frac{1}{\lfloor\underline{-1}} - \frac{1}{\lfloor\underline{0}}$$

$$t_2 = \frac{1}{\lfloor\underline{2-2}} + \frac{1}{\lfloor\underline{2-1}} = \frac{1}{\lfloor\underline{0}} - \frac{1}{\lfloor\underline{1}}$$

$$t_3 = \frac{1}{\lfloor\underline{3-2}} + \frac{1}{\lfloor\underline{3-1}} = \frac{1}{\lfloor\underline{1}} - \frac{1}{\lfloor\underline{2}}$$

$$t_4 = \frac{1}{\lfloor\underline{4-2}} + \frac{1}{\lfloor\underline{4-1}} = \frac{1}{\lfloor\underline{2}} - \frac{1}{\lfloor\underline{3}}$$

……………………………………………………………………………………

এই পদগুলো যোগ করে পাই,

S= $t_1 + t_2 + t_3 + t_4 + \ldots\ldots\ldots\ldots\ldots$

$$= \left(\frac{1}{\lfloor -1} + \frac{1}{\lfloor 0} + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \cdots.\right) + \left(\frac{1}{\lfloor 0} + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \ldots\right.$$

$$= \left(\frac{1}{\infty} + \frac{1}{1} + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \cdots.\right) + \left(\frac{1}{1} + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \ldots.\right)$$

$$= \left(0 + 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \ldots\right) + \left(\frac{1}{1} + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \ldots.\right)$$

= e+ e = 2e Proved