সূচক ধারা

EXPONENTIAL SERIES

CH 4

1) $$e^{x} = 1 + \frac{x}{\underline{\lfloor 1}} + \frac{x^2}{\underline{\lfloor 2}} + \frac{x^3}{\underline{\lfloor 3}} + \frac{x^4}{\underline{\lfloor 4}} \ldots\ldots\ldots\ldots\ldots.$$

2) $$e^{-x} = 1 - \frac{x}{\underline{\lfloor 1}} + \frac{x^2}{\underline{\lfloor 2}} - \frac{x^3}{\underline{\lfloor 3}} + \frac{x^4}{\underline{\lfloor 4}} - \ldots\ldots\ldots\ldots\ldots.$$

3) $$\frac{e^{x} + e^{-x}}{2} = \left(1 + \frac{x^2}{\underline{\lfloor 2}} + \frac{x^4}{\underline{\lfloor 4}} + \frac{x^6}{\underline{\lfloor 6}} + \ldots\ldots\ldots\right)$$

4) $$\frac{e^{x} - e^{-x}}{2} = \left(\frac{x}{\underline{\lfloor 1}} + \frac{x^3}{\underline{\lfloor 3}} + \frac{x^5}{\underline{\lfloor 5}} + \frac{x^7}{\underline{\lfloor 7}} + \ldots\ldots\ldots\right)$$

5) $$\mathrm{e} = 1 + \frac{1}{\underline{\lfloor 1}} + \frac{1}{\underline{\lfloor 2}} + \frac{1}{\underline{\lfloor 3}} + \ldots\ldots\ldots.\ldots\infty$$

6) $$e^{-1} = 1 - \frac{1}{\underline{\lfloor 1}} + \frac{1}{\underline{\lfloor 2}} - \frac{1}{\underline{\lfloor 3}} + \ldots\ldots\ldots.\ldots\infty$$

7) $\dfrac{e^{1} + e^{-1}}{2} = \left(1 + \dfrac{1}{\underline{\lfloor 2}} + \dfrac{1}{\underline{\lfloor 4}} + \dfrac{1}{\underline{\lfloor 6}} + \ldots\ldots\ldots \right)$

8) $\dfrac{e^{1} - e^{-1}}{2} = \left(1 + \dfrac{1}{\underline{\lfloor 3}} + \dfrac{1}{\underline{\lfloor 5}} + \dfrac{1}{\underline{\lfloor 7}} + \ldots\ldots\ldots \right)$

9) $a^{x} = 1 + \dfrac{(\text{x}\log a)}{\underline{\lfloor 1}} + \dfrac{(x \log a)^{2}}{\underline{\lfloor 2}} + \dfrac{(x \log a)^{3}}{\underline{\lfloor 3}} + \ldots\ldots\ldots$

10) $a^{-x} = 1 - \dfrac{(\text{x}\log a)}{\underline{\lfloor 1}} + \dfrac{(x \log a)^{2}}{\underline{\lfloor 2}} - \dfrac{(x \log a)^{3}}{\underline{\lfloor 3}} + \ldots \ldots \ldots .$

6) দেখাও যে, $\frac{1}{\lfloor 2} + \frac{3}{\lfloor 4} + \frac{5}{\lfloor 6} + \cdots \ldots \ldots \ldots \ldots \ldots = 1 - \frac{1}{e}$

$$\text{L.H.S} = \frac{1}{\lfloor 2} + \frac{3}{\lfloor 4} + \frac{5}{\lfloor 6} + \cdots \ldots \ldots \ldots \ldots \ldots$$

ধারাটির n - তম পদ $= \frac{\{1+(n-1)2\}}{\lfloor 2+(n-1)2}$

$$= \frac{(1+2n-2)}{\lfloor 2+2n-2}$$

$$= \frac{(2n-1)}{\lfloor 2n}$$

$$= \frac{2n}{\lfloor 2n} - \frac{1}{\lfloor 2n}$$

$$= \frac{2n}{2n\lfloor (2n-1)} - \frac{1}{\lfloor 2n} = \frac{1}{\lfloor 2n-1} - \frac{1}{\lfloor 2n}$$

ধারাটির n - তম পদ $= \dfrac{1}{\lfloor\underline{2n-1}} - \dfrac{1}{\lfloor\underline{2n}}$

n এর পরিবর্তে 1,2,3,....... ......ইতাদি বসাইয়া পাই,

$$t_1 = \frac{1}{\lfloor\underline{2-1}} - \frac{1}{\lfloor\underline{2}} = \frac{1}{\lfloor\underline{1}} - \frac{1}{\lfloor\underline{2}}$$

$$t_2 = \frac{1}{\lfloor\underline{4-1}} - \frac{1}{\lfloor\underline{4}} = \frac{1}{\lfloor\underline{3}} - \frac{1}{\lfloor\underline{4}}$$

$$t_3 = \frac{1}{\lfloor\underline{6-1}} - \frac{1}{\lfloor\underline{6}} = \frac{1}{\lfloor\underline{5}} - \frac{1}{\lfloor\underline{6}}$$

$$t_4 = \frac{1}{\lfloor\underline{8-1}} - \frac{1}{\lfloor\underline{8}} = \frac{1}{\lfloor\underline{7}} - \frac{1}{\lfloor\underline{8}}$$

....

এই পদগুলো যোগ করে পাই,

$$S = t_1 + t_2 + t_3 + t_4 + \dots\dots\dots\dots\dots$$

$$= \left(\frac{1}{\lfloor 1} + \frac{1}{\lfloor 3} + \frac{1}{\lfloor 5} + \frac{1}{\lfloor 7} + \cdots.\right) - \left(\frac{1}{\lfloor 2} + \frac{1}{\lfloor 4} + \frac{1}{\lfloor 6} + \frac{1}{\lfloor 8} + \dots\right.$$

$$= \left(\frac{1}{\lfloor 1} + \frac{1}{\lfloor 3} + \frac{1}{\lfloor 5} + \frac{1}{\lfloor 7} + \cdots.\right) - \left(1 + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 4} + \frac{1}{\lfloor 6} + \frac{1}{\lfloor 8} + \dots\dots -1\right)$$

$$= \frac{e^1 - e^{-1}}{2} + \frac{e^1 + e^{-1}}{2} - 1$$

$$= \frac{e^1 - e^{-1} + e^1 + e^{-1} - 2}{2} = \frac{2e^1 - 2}{2} = e^1 - 1$$

7(vi)

দেখাও যে, $1+\frac{1+2}{\lfloor 2}+\frac{1+2+2^2}{\lfloor 3}+\frac{1+2+2^2+2^3}{\lfloor 4}+.\ldots\ldots = e^2-1$

$$\text{L.H.S} = 1+\frac{1+2}{\lfloor 2}+\frac{1+2+2^2}{\lfloor 3}+\frac{1+2+2^2+2^3}{\lfloor 4}+.\ldots\ldots$$

ধারাটির n - তম পদ $=\frac{1+2+2^2+2^3+\cdots\ldots\ldots+2^{n-1}}{\lfloor n}$

$$=\frac{1.\frac{2^n-1}{2-1}}{\lfloor n}$$

$$=\frac{2^n-1}{\lfloor n}$$

ধারাটির n - তম পদ $= \frac{2^n - 1}{\lfloor\underline{n}} = \frac{2^n}{\lfloor\underline{n}} - \frac{1}{\lfloor\underline{n}}$

n এর পরিবর্তে 1,2,3,……. ……ইতাদি বসাইয়া পাই,

$$t_1 = \frac{2^1}{\lfloor\underline{1}} - \frac{1}{\lfloor\underline{1}}$$

$$t_2 = \frac{2^2}{\lfloor\underline{2}} - \frac{1}{\lfloor\underline{2}}$$

$$t_3 = \frac{2^3}{\lfloor\underline{3}} - \frac{1}{\lfloor\underline{3}}$$

$$t_4 = \frac{2^4}{\lfloor\underline{4}} - \frac{1}{\lfloor\underline{4}}$$

....

এই পদগুলো যোগ করে পাই,

S= $t_1 + t_2 + \ t_3 + t_4 + \ \ldots\ldots\ldots\ldots\ldots$

$$= \left(\frac{2^1}{\lfloor\underline{1}} + \frac{2^2}{\lfloor\underline{2}} + \frac{2^3}{\lfloor\underline{3}} + \frac{2^4}{\lfloor\underline{4}} + \ldots\ldots.. \right) - \left(\frac{1}{\lfloor\underline{1}} + \frac{1}{\lfloor\underline{2}} + \frac{1}{\lfloor\underline{3}} + \frac{1}{\lfloor\underline{4}} + \ldots\ldots.. \right)$$

$$= \left(1 + \frac{2^1}{\lfloor\underline{1}} + \frac{2^2}{\lfloor\underline{2}} + \frac{2^3}{\lfloor\underline{3}} + \ldots.. - 1\right) - \left(1 + \frac{1}{\lfloor\underline{1}} + \frac{1}{\lfloor\underline{2}} + \frac{1}{\lfloor\underline{3}} + \ \ldots.. - 1\right)$$

$$= \ (e^2 - 1\ ) - (e - 1\ )$$

$$= \ (e^2 - 1\ - e + 1\ )$$

$$= \ (e^2 - e\ )$$

গুণোত্তর ধারার ক্ষেত্রে :

$$1 + 2 + 2^2 + 2^3 + \ldots\ldots\ldots$$

ধারাটির যোগফল:

$$= a\,\frac{r^n - 1}{r - 1}$$

$$= 1\,\frac{2^n - 1}{2 - 1}$$

$$= 2^n - 1$$

1) $e^x = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \frac{x^4}{\lfloor 4} \ldots \ldots \ldots \ldots \ldots .$

$$e^2 = 1 + \frac{2}{\lfloor 1} + \frac{2^2}{\lfloor 2} + \frac{2^3}{\lfloor 3} + \frac{2^4}{\lfloor 4} \ldots \ldots \ldots \ldots \ldots .$$

প্রমাণ কর যে, $2 < e < 3$

সমাধান: আমরা জানি

$$e^1 = 1 + \frac{1}{\lfloor 1} + \frac{1^2}{\lfloor 2} + \frac{1^3}{\lfloor 3} + \ldots\ldots\ldots\ldots\ldots..\infty$$

$$= 1 + 1 + \frac{1^2}{\lfloor 2} + \frac{1^3}{\lfloor 3} + \ldots\ldots\ldots\ldots\ldots..\infty$$

$$= 2 + \frac{1^2}{\lfloor 2} + \frac{1^3}{\lfloor 3} + \ldots\ldots\ldots\ldots\ldots..\infty$$

সুতরাং $e > 2$

আবার, $\underline{|3} = 3.2.1$

$1.2.3 > 2^2$

$$\therefore \frac{1}{\underline{|3}} < \frac{1}{2^2}$$

আবার, $\underline{|4} = 4.3.2.1$

$1.2.3.4 > 2^3$

$$\therefore \frac{1}{\underline{|4}} < \frac{1}{2^3}$$

অনুরূপভাবে, $\frac{1}{\lfloor\underline{5}} < \frac{1}{2^4}, \frac{1}{\lfloor\underline{6}} < \frac{1}{2^5}$ ইত্যাদি।

$$e = 1 + \frac{1}{\lfloor\underline{1}} + \frac{1^2}{\lfloor\underline{2}} + \frac{1^3}{\lfloor\underline{3}} + \frac{1^4}{\lfloor\underline{4}} + \cdots \ldots\ldots\ldots\ldots\ldots\ldots \infty$$

$$e < 1 + \left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\ldots\ldots\ldots\right)$$

$$e < 1 + 1\, \frac{1 - \left(\frac{1}{2}\right)^{\infty}}{1 - \frac{1}{2}}$$

$$< 1 + 1\, \frac{1-0}{\frac{1}{2}}$$

$$< 1 + 1.2$$

$$e < 3$$

সুতরাং e এর মান 3 অপেক্ষা ক্ষুদ্রতর এবং 2 অপেক্ষা বৃহত্তর।