

$$4. i) \quad \sin^2 \frac{17\pi}{18} + \sin^2 \frac{5\pi}{8} + \cos^2 \frac{37\pi}{18} + \cos^2 \frac{3\pi}{8}$$

$$= \sin^2 \left(\pi - \frac{\pi}{18} \right) + \sin^2 \left(\pi - \frac{3\pi}{8} \right) + \cos^2 \left(2\pi + \frac{\pi}{18} \right) + \cos^2 \frac{3\pi}{8}$$

$$= \sin^2 \frac{\pi}{18} + \sin^2 \frac{3\pi}{8} + \cos^2 \frac{\pi}{18} + \cos^2 \frac{3\pi}{8}$$

$$= \sin^2 \frac{\pi}{18} + \cos^2 \frac{\pi}{18} + \sin^2 \frac{3\pi}{8} + \cos^2 \frac{3\pi}{8}$$

$$= 1 + 1 = 2$$

4.ii)

$$\begin{aligned} & \sin^2 \frac{\pi}{7} + \sin^2 \frac{5\pi}{14} + \sin^2 \frac{8\pi}{7} + \sin^2 \frac{9\pi}{14} \\ &= \sin^2 \frac{\pi}{7} + \sin^2 \left(\frac{\pi}{2} - \frac{\pi}{7} \right) + \sin^2 \left(\pi + \frac{\pi}{7} \right) + \sin^2 \left(\frac{\pi}{2} + \frac{\pi}{7} \right) \\ &= \sin^2 \frac{\pi}{7} + \sin^2 \left(\frac{\pi}{2} - \frac{\pi}{7} \right) + \sin^2 \left(\pi + \frac{\pi}{7} \right) + \sin^2 \left(\frac{\pi}{2} + \frac{\pi}{7} \right) \\ &= \sin^2 \frac{\pi}{7} + \cos^2 \frac{\pi}{7} + \sin^2 \frac{\pi}{7} + \cos^2 \frac{\pi}{7} \\ &= 1 + 1 = 2 \text{ Ans.} \end{aligned}$$

4.iii)

$$\begin{aligned} & \cos^2 \frac{\pi}{24} + \cos^2 \frac{19\pi}{24} + \cos^2 \frac{31\pi}{24} + \cos^2 \frac{37\pi}{24} \\ &= \cos^2 \frac{\pi}{24} + \cos^2 \left(\pi - \frac{5\pi}{24} \right) + \cos^2 \left(\frac{3\pi}{2} - \frac{5\pi}{24} \right) + \cos^2 \left(\frac{3\pi}{2} + \frac{\pi}{24} \right) \\ &= \cos^2 \frac{\pi}{24} + \cos^2 \frac{5\pi}{24} + \sin^2 \frac{5\pi}{24} + \sin^2 \frac{\pi}{24} \\ &= 1 + 1 = 2 \text{ Ans.} \end{aligned}$$

$$4.vii) \quad \cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \frac{5\pi}{8} + \cos^2 \frac{7\pi}{8}$$

$$= \cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \left(\pi - \frac{3\pi}{8} \right) + \cos^2 \left(\pi - \frac{\pi}{8} \right)$$

$$= \cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \frac{\pi}{8}$$

$$= 2 \left(\cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} \right)$$

$$= 2 \left\{ \cos^2 \frac{\pi}{8} + \cos^2 \left(\frac{\pi}{2} - \frac{\pi}{8} \right) \right\}$$

$$= 2 \left(\cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8} \right)$$

$$= 1 + 1 = 2$$

$$4.viii) \sin^2 \frac{\pi}{4} + \sin^2 \frac{3\pi}{4} + \sin^2 \frac{5\pi}{4} + \sin^2 \frac{7\pi}{4}$$

$$= \sin^2 \frac{\pi}{4} + \sin^2 \left(\pi - \frac{\pi}{4} \right) + \sin^2 \left(\pi - \frac{\pi}{4} \right) + \sin^2 \left(2\pi - \frac{\pi}{4} \right)$$

$$= \sin^2 \frac{\pi}{4} + \sin^2 \frac{\pi}{4} + \sin^2 \frac{\pi}{4} + \sin^2 \frac{\pi}{4}$$

$$= 4 \sin^2 \frac{\pi}{4}$$

$$= 4 \left(\sin \frac{\pi}{4} \right)^2$$

$$= 4 (\sin 45^\circ)^2$$

$$= 4 \left(\frac{1}{\sqrt{2}} \right)^2$$

$$= 4 \frac{1}{2} = 2 \text{ Ans.}$$