

TRIGONOMETRICAL RATIOS OF COMPOUND ANGLES

$$1) \sin (A + B) = \sin A \cdot \cos B + \cos A \cdot \sin B$$

$$2) \sin (A - B) = \sin A \cdot \cos B - \cos A \cdot \sin B$$

$$3) \cos (A + B) = \cos A \cdot \cos B - \sin A \cdot \sin B$$

$$4) \cos (A - B) = \cos A \cdot \cos B + \sin A \cdot \sin B$$

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A . \tan B}$$

$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A . \tan B}$$

$$\cot (A + B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

$$\cot (A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

প্র-১৩। $\cos 75^\circ$ এর মান কত?

সমাধান : $\cos 75^\circ = \cos (45^\circ + 30^\circ)$
 $= \cos 45^\circ \cdot \cos 30^\circ - \sin 45^\circ \cdot \sin 30^\circ$
 $= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3} - 1}{2\sqrt{2}} = \frac{\sqrt{2}(\sqrt{3} - 1)}{2\sqrt{2} \cdot \sqrt{2}} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$
[বাকী]

প্র-১৪। $\cot 75^\circ$ এর মান নির্ণয় কর।

সমাধান : $\cot 75^\circ = \cot (45^\circ + 30^\circ)$
 $= \frac{\cot 45^\circ \cdot \cot 30^\circ - 1}{\cot 45^\circ + \sec 30^\circ}$
 $= \frac{1 \cdot \sqrt{3} - 1}{1 + \sqrt{3}}$

প্র-১৫। $\cos^2 30^\circ - \sin^2 30^\circ$ এর মান কত?

$$\begin{aligned}\text{সমাধান : } & \cos^2 30^\circ - \sin^2 30^\circ \\ &= \cos (30^\circ + 30^\circ) \cos (30^\circ - 30^\circ) \\ &= \cos 60^\circ \cos 0^\circ \\ &= \frac{1}{2} \times 1 = \frac{1}{2}\end{aligned}$$

প্র-১৬। $1 - \cos A \cos B + \sin A \sin B = 0$ হলে, $A + B$ এর মান কত?

$$\text{সমাধান : } 1 - \cos A \cos B + \sin A \sin B = 0$$

$$\text{বা, } \cos A \cos B - \sin A \sin B = 1$$

$$\text{বা, } \cos (A + B) = \cos 0^\circ$$

$$\text{বা, } A + B = 0^\circ \text{ (Ans.)}$$

প্রশ্ন-১৭। প্রমাণ কর যে, $\cos 81^\circ 26' \cos 21^\circ 26' + \cos 8^\circ 34' \cos 68^\circ 34' = \frac{1}{2}$.

সমাধান : $\cos 81^\circ 26' \cos 21^\circ 26' + \cos(90^\circ - 81^\circ 26') \cos(90^\circ - 21^\circ 26')$

$$= \cos 81^\circ 26' \cos 21^\circ 26' + \sin 81^\circ 26' \sin 21^\circ 26'$$

$$= \cos (81^\circ 26' - 21^\circ 26')$$

$$= \cos 60^\circ$$

$$= \frac{1}{2} = \text{ডানপক্ষ}$$

$\therefore$ বামপক্ষ = ডানপক্ষ (প্রমাণিত)

প্রশ্ন-১৮। $\sin(A - B)$ এর সূত্রটি লিখ।

সমাধান : $\sin(A - B) = \sin A \cos B - \cos A \sin B$.

প্রশ্ন-১৯। যদি $\cos A = \frac{4}{5}$, $\cos B = \frac{3}{5}$ হয়, তবে $\cos(A + B)$ এর মান কত?

$$\begin{aligned} \text{উত্তর} \quad \sin A &= \sqrt{1 - \cos^2 A} = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{25 - 16}{25}} \\ &= \sqrt{\frac{9}{25}} = \frac{3}{5} \end{aligned}$$

$$\sin B = \sqrt{1 - \cos^2 B} = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{25 - 9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\therefore \cos(A + B) = \cos A \cos B - \sin A \sin B = \frac{4}{5} \cdot \frac{3}{5} - \frac{3}{5} \cdot \frac{4}{5} = \frac{12}{25} - \frac{12}{25} = 0 \text{ Ans.}$$