TRIGONOMETRICAL RATIOS OF COMPOUND ANGLES

1) $\sin(A + B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

2) $\sin(A - B) = \sin A \cdot \cos B - \cos A \cdot \sin B$

3) $\cos(A + B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

4) $\cos(A - B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

tan (A + B) = $\dfrac{tanA + tanB}{1\ - tanA.tanB}$

tan (A - B) = $\dfrac{tanA - tanB}{1 + tanA.tanB}$

$$\cot(A + B) = \frac{cotA\ cotB - 1}{cotB + cotA}$$

$$\cot(A - B) = \frac{cotA\ cotB + 1}{cotB - cotA}$$

1. মান নির্ণয় কর:

(i) $\sin 105^\circ$

$$= \sin(60^\circ + 45^\circ)$$

$$= \sin 60^\circ \cdot \cos 45^\circ + \cos 60^\circ \cdot \sin 45^\circ$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}} \text{ Ans.}$$

২। মান নির্ণয় কর:

(ii) $\cos 68^\circ 20' \cdot \cos 8^\circ 20' + \cos 81^\circ 40' \cdot \cos 21^\circ 40'$

$$= \cos(90^\circ - 21^\circ 40') \cdot \cos 8^\circ 20' + \cos(90^\circ - 8^\circ 20') \cdot \cos 21^\circ 40'$$

$$= \sin 21^\circ 40' \cdot \cos 8^\circ 20' + \sin 8^\circ 20' \cdot \cos 21^\circ 40'$$

$$= \sin(21^\circ 40' + 8^\circ 20')$$

$$= \sin 30^\circ$$

$$= \frac{1}{2} \text{ Ans.}$$

প্রশ্ন-৩। যদি $A + B = \frac{\pi}{4}$ হয় তবে দেখাও যে, $(1 + \tan A)(1 + \tan B) = 2$।

সমাধান : দেওয়া আছে $A+B=\frac{\pi}{4}$

$$\Rightarrow \tan(A+B) = \tan\frac{\pi}{4}$$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = 1$$

$$\Rightarrow \tan A + \tan B = 1 - \tan A \cdot \tan B$$

$$\Rightarrow \tan A + \tan B + \tan A \cdot \tan B = 1$$

$$\Rightarrow \tan A + \tan B + \tan A \cdot \tan B + 1 = 1 + 1$$

$$\Rightarrow \tan A + \tan A \cdot \tan B + \tan B + 1 = 2$$

$$\Rightarrow \tan A(1 + \tan B) + 1(1 + \tan B) = 2$$

$$\Rightarrow (1 + \tan B)(1 + \tan A) = 2 \text{ proved.}$$

প্রশ্ন-৬। যদি $\sin\alpha\ \sin\beta - \cos\alpha\ \cos\beta + 1 = 0$ হয় তবে দেখাও যে, $1 + \cot\alpha\ .\ \tan\beta=0$ অথবা, $1+\tan\alpha\ \cot\beta= 0$

[বাকাশিবো–২০০৬, '১৪,'১৭R]

সমাধান:

$$\sin\alpha . \sin\beta - \cos\alpha . \cos\beta + 1 = 0$$

$$\Rightarrow 1 = \cos\alpha . \cos\beta - \sin\alpha . \sin\beta$$

$$\Rightarrow 1 = \cos(\alpha+\beta)$$

$$\Rightarrow 1^2 = \cos^2(\alpha+\beta)$$

$$\Rightarrow 1 - \cos^2(\alpha+\beta) = 0$$

$$\Rightarrow \sin^2(\alpha+\beta) = 0$$

$$\Rightarrow \sin(\alpha+\beta) = 0$$

$$\Rightarrow \sin\alpha.\cos\beta + \cos\alpha.\sin\beta = 0 \quad \ldots\ldots (i)$$

উভয় পক্ষকে $\sin\alpha.\cos\beta$ দ্বারা ভাগ করিয়া পাই,

$$\Rightarrow 1 + \cot\alpha.\tan\beta = 0 \text{ proved.}$$

আবার (i) নং সমীকরণকে $\cos\alpha.\sin\beta$ দ্বারা ভাগ করিয়া পাই,

$$\tan\alpha.\cot\beta + 1 = 0$$

$$\Rightarrow 1 + \tan\alpha.\cot\beta = 0 \text{ proved.}$$