TRIGONOMETRICAL RATIOS OF COMPOUND ANGLES

1) $\sin(A + B) = \sin A \,.\, \cos B + \cos A \,.\, \sin B$

2) $\sin(A - B) = \sin A \,.\, \cos B - \cos A \,.\, \sin B$

3) $\cos(A + B) = \cos A \,.\, \cos B - \sin A \,.\, \sin B$

4) $\cos(A - B) = \cos A \,.\, \cos B + \sin A \,.\, \sin B$

tan (A + B) = $\dfrac{tanA + tanB}{1 - tanA.tanB}$

tan (A - B) = $\dfrac{tanA - tanB}{1 + tanA.tanB}$

$$\cot(A + B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

$$\cot(A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

প্রশ্ন-৭। যদি $\sin\alpha \cos\beta - \cos\alpha \sin\beta = 1$ হয় তবে দেখাও যে, $1 + \tan\alpha . \tan\beta = 0$ অথবা $1 + \cot\alpha \cot\beta = 0$

[বাকাশিবো–২০০৮, '০৯, '১১]

সমাধান:

$$\sin\alpha . \cos\beta - \cos\alpha . \sin\beta = 1$$

$$\Rightarrow \sin(\alpha - \beta) = 1$$

$$\Rightarrow \sin^2(\alpha - \beta) = 1$$

$$\Rightarrow 1 - \sin^2(\alpha - \beta) = 0$$

$$\Rightarrow \cos^2(\alpha - \beta) = 0$$

$$\Rightarrow \cos(\alpha - \beta) = 0$$

$\Rightarrow \cos\alpha.\cos\beta + \sin\alpha.\sin\beta = 0$ ---------- (i)

উভয় পক্ষে $\cos\alpha.\cos\beta$ দ্বারা ভাগ করে পাই,

$$\frac{\cos\alpha.\cos\beta}{\cos\alpha.\cos\beta} + \frac{\sin\alpha.\sin\beta}{\cos\alpha.\cos\beta} = 0$$

$\Rightarrow 1 + \tan\alpha.\tan\beta = 0$ proved.

উভয় পক্ষে $\sin\alpha.\sin\beta$ দ্বারা ভাগ করে পাই,

$$\frac{\cos\alpha.\cos\beta}{\sin\alpha.\sin\beta} + \frac{\sin\alpha.\sin\beta}{\sin\alpha.\sin\beta} = 0$$

$\Rightarrow \cot\alpha.\cot\beta + 1 = 0$ proved.

প্রশ্ন–৮। যদি $\tan\alpha + \tan\beta = y$, $\cot\alpha + \cot\beta = x$ এবং $\alpha + \beta = \theta$ হয়,

তবে প্রমাণ কর যে, $(x - y)\tan\theta = xy$ [বাকাশিবো–২০০৪, ;১৪, '১৪T]

অথবা, $\tan\alpha + \tan\beta = b$, $\cot\alpha + \cot\beta = a$, এবং $\alpha + \beta = \theta$ হয়,

তবে দেখাও যে, $(a - b)\tan\theta = ab$ [বাকাশিবো–২০০৪,'০৭,'১১T,'১১R,'১২,'১৪,'১৪R,'১৫,'১৫R,'১৭,'১৮]

সমাধান : দেওয়া আছে, $\cot\alpha + \cot\beta = a$

$\tan\alpha + \tan\beta = b$ এবং $\alpha + \beta = \theta$

এখানে, $\cot\alpha + \cot\beta = a$

$$\Rightarrow \frac{1}{\tan\alpha} + \frac{1}{\tan\beta} = a$$

$$\Rightarrow \frac{\tan\beta + \tan\alpha}{\tan\alpha \cdot \tan\beta} = a$$

$$\Rightarrow \frac{b}{\tan\alpha \cdot \tan\beta} = a$$

$$\therefore \tan\alpha \cdot \tan\beta = \frac{b}{a}$$

এবং $\alpha + \beta = \theta$

$$\Rightarrow \tan(\alpha + \beta) = \tan\theta$$

$$\Rightarrow \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta} = \tan\theta$$

$$\Rightarrow \frac{b}{1 - \frac{b}{a}} = \tan\theta$$

$$\Rightarrow \frac{b}{\frac{a - b}{a}} = \tan\theta$$

$$\Rightarrow \frac{ab}{a - b} = \tan\theta$$

$$\therefore (a - b)\tan\theta = ab$$

অথবা, $(a - b) = ab \cdot \frac{1}{\tan\theta}$

$\therefore (a - b) = ab \cot\theta$ **(Proved).**

প্রশ্ন-১০। যদি $\alpha + \beta = \theta$ এবং $\tan\alpha = k \tan\beta$ হয় তবে প্রমাণ কর যে, $\sin(\alpha - \beta) = \frac{k - 1}{k + 1} \sin\theta$

সমাধান

দেওয়া আছে, $\alpha + \beta = \theta \cdots\cdots (i)$

এবং $\tan\alpha = k \tan\beta$

$$\Rightarrow \frac{\tan\alpha}{\tan\beta} = \frac{k}{1}$$

$$\Rightarrow \frac{\tan\alpha - \tan\beta}{\tan\alpha + \tan\beta} = \frac{k-1}{k+1}$$ বিয়োজন যোজন করে পাই,

$$\Rightarrow \frac{\frac{\sin\alpha}{\cos\alpha} - \frac{\sin\beta}{\cos\beta}}{\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}} = \frac{k-1}{k+1}$$

$$\Rightarrow \frac{\dfrac{\sin\alpha.\cos\beta - \sin\beta.\cos\alpha}{\cos\alpha.\cos\beta}}{\dfrac{\sin\alpha.\cos\beta + \cos\alpha.\sin\beta}{\cos\alpha.\cos\beta}} = \frac{K-1}{K+1}$$

$$\Rightarrow \frac{\dfrac{\sin(\alpha-\beta)}{\cos\alpha.\cos\beta}}{\dfrac{\sin(\alpha+\beta)}{\cos\alpha.\cos\beta}} = \frac{K-1}{K+1}$$

$$\Rightarrow \frac{\sin(\alpha-\beta)}{\cos\alpha.\cos\beta} \times \frac{\cos\alpha.\cos\beta}{\sin(\alpha+\beta)} = \frac{K-1}{K+1}$$

$$\Rightarrow \frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)} = \frac{k-1}{k+1}$$

$$\Rightarrow \frac{\sin(\alpha-\beta)}{\sin\theta} = \frac{k-1}{k+1}$$

$$\Rightarrow \sin(\alpha-\beta) = \frac{k-1}{k+1} \cdot \sin\theta$$ proved.