$\sin(A + B) = \sin A \cdot \cos B + \cos A \sin B$.....(i)

$\sin(A - B) = \sin A \cdot \cos B - \cos A \sin B$ .....(ii)

$\cos(A + B) = \cos A \cdot \cos B + \sin A \sin B$ .....(iii)

$\cos(A - B) = \cos A \cdot \cos B - \sin A \sin B$ .....(iv)

(i) + (ii) হতে পাই,

$\sin(A + B) = \sin A \cdot \cos B + \cos A \sin B$.....(i)

$\sin(A - B) = \sin A \cdot \cos B - \cos A \sin B$ .....(ii)

$\sin(A+B) + \sin(A-B) = 2\sin A \cos B$

$$\sin(A + B) = \sin A.\cos B + \cos A \sin B \ldots\ldots\ldots (i)$$

(i) - (ii) হতে পাই,

$$\sin(A - B) = \sin A.\cos B - \cos A \sin B \ldots\ldots\ldots (ii)$$

$$\sin(A+B) - \sin(A-B) = 2\cos A \sin B$$

(iii) + (iv) হতে পাই,

$$\cos(A + B) = \cos A.\cos B - \sin A \sin B \ldots\ldots\ldots (iii)$$

$$\cos(A - B) = \cos A.\cos B + \sin A \sin B \ldots\ldots\ldots (iv)$$

$$\cos(A + B) + \cos(A - B) = 2\cos A \cos B$$

(iii) - (iv) হতে পাই,

$$\cos(A + B) = \cos A.\cos B - \sin A \sin B \ldots\ldots\ldots (iii)$$

$$\cos(A - B) = \cos A.\cos B + \sin A \sin B \ldots\ldots\ldots (iv)$$

$$\cos(A + B) - \cos(A - B) = -2\sin A \sin B$$

$$\Rightarrow \quad \cos(A - B) - \cos(A + B) = 2\sin A \sin B$$

ধরি $A + B = C$ এবং $A - B = D$

$2A = C + D$

$\Rightarrow A = \frac{C+D}{2}$

$2B = C - D$

$\Rightarrow B = \frac{C-D}{2}$

$\sin(A+B) + \sin(A-B) = 2\sin A \cos B$

$\Rightarrow \sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$

$\sin(A+B) - \sin(A-B) = 2\cos A \sin B$

$\Rightarrow \sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$

$\cos(A + B) + \cos(A - B) = 2\cos A \cos B$

$\Rightarrow \cos C + \cos D = 2\cos \frac{C+D}{2} \cos \frac{C-D}{2}$

$\cos(A - B) - \cos(A + B) = 2\sin A \sin B$

$\Rightarrow \cos D - \cos C = 2\sin \frac{C+D}{2} \sin \frac{C-D}{2}$

$\Rightarrow \cos C - \cos D = 2\sin \frac{C+D}{2} \sin \frac{D-C}{2}$

.1.	$\sin(A+B)+\sin(A-B)=2\sin A\cos B$	5.	$\sin(A+B)-\sin(A-B)=2\cos A\sin B$
2.	$\cos(A+B)+\cos(A-B)=2\cos A\cos B$	6.	$\cos(A-B)-\cos(A+B)=2\sin A\sin B$
3.	$\sin C+\sin D=2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$	7.	$\sin C-\sin D=2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$
4.	$\cos C+\cos D=2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$	8.	$\cos C-\cos D=2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$

1.

iii) Sin $55° + \cos 55°$

$= \text{Sin}\,(90° - 35°) + \cos 55°$

$= \cos 35° + \cos 55°$

$= \cos 55° + \cos 35°$

$= 2cos\dfrac{35° + 55°}{2} \cos\dfrac{55° - 35°}{2}$

$= 2cos\dfrac{90°}{2} \cos\dfrac{20°}{2}$

$= 2cos45° \cos 10°$

$= 2\dfrac{1}{\sqrt{2}}\cos 10°$

$= \sqrt{2}\cos 10°$

Formula

cos C + cos D = $2\cos\frac{C+D}{2} \cos\frac{C-D}{2}$

2. sinA + sinB = cosA + cosB

$\Rightarrow 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$

$\Rightarrow 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2} = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$

$\Rightarrow \frac{\sin \frac{A+B}{2}}{\cos \frac{A+B}{2}} = 1$

$\Rightarrow \tan \frac{A+B}{2} = 1$

$\Rightarrow \tan \frac{A+B}{2} = \tan \frac{\pi}{4}$

$\Rightarrow \frac{A+B}{2} = \frac{\pi}{4}$

$\Rightarrow A + B = \frac{\pi}{2}$ *Proved*

Formula

i) $\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$

ii) $\cos C + \cos D = 2\cos \frac{C+D}{2} \cos \frac{C-D}{2}$

3.

L.H.S.

$$= \text{Cot}\,(A + 15°) - \tan(A - 15°)$$

$$= \frac{\cos(A + 15°)}{\sin(A + 15°)} - \frac{\sin(A - 15°)}{\text{Cos}\,(A - 5°)}$$

$$= \frac{\cos(A + 15°)\cos(A - 15°) - \sin(A + 15°)\ \sin(A - 15°)}{\sin(A + 15°)\cos(A - 15°)}$$

$$= \frac{\cos(A + 15° + A - 15°)}{\sin(A + 15°)\cos(A - 15°)}$$

$$= \frac{\cos\ 2A}{\frac{1}{2}\ 2\sin(A + 15°)\cos(A - 15°)}$$

Formula

(i) cos (A + B) = cosA. cosB - sinA sinB

$$= \frac{2\cos 2A}{2\sin(A+15°)\cos(A-15°)}$$

$$= \frac{2\cos 2A}{\sin(A+15°+A+15°)+\sin(A+15°-A+15°)}$$

$$= \frac{2\cos 2A}{\sin(A+15°+A-15°)+\sin(A+15°-A+15°)}$$

$$= \frac{2\cos 2A}{\sin 2A+\sin 30°}$$

$$= \frac{2\cos 2A}{\sin 2A+\frac{1}{2}}$$

$$= \frac{2\cos 2A}{\frac{2sinA+1}{2}}$$

$$= \frac{4\cos 2A}{2sinA+1} \quad proved$$

Formula
(i) Sin (A+B) + sin (A-B) = 2sinA cosB

4. $2\cos\frac{\pi}{7}\cos\frac{3\pi}{7}+\cos\frac{3\pi}{7}+\cos\frac{5\pi}{7}$

$$=2\cos\frac{\pi}{7}\cos\frac{3\pi}{7}+2\cos\frac{\frac{5\pi}{7}+\frac{3\pi}{7}}{2}\cos\frac{\frac{5\pi}{7}-\frac{3\pi}{7}}{2}$$

$$=2\cos\frac{\pi}{7}\cos\frac{3\pi}{7}+2\cos\frac{\frac{8\pi}{7}}{2}\cos\frac{\frac{2\pi}{7}}{2}$$

$$=2\cos\frac{\pi}{7}\cos\frac{3\pi}{7}+2\cos\frac{4\pi}{7}\cos\frac{\pi}{7}$$

$$=2\cos\frac{\pi}{7}\cos\frac{3\pi}{7}+2\cos(\pi-\frac{3\pi}{7})\cos\frac{\pi}{7}$$

$$=2\cos\frac{\pi}{7}\cos\frac{3\pi}{7}-2\cos\frac{3\pi}{7}\cos\frac{\pi}{7}$$

$$=0=R.H.S.$$

Formula

$\cos C+\cos D=2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

5. L.H.S.

$$tan\frac{45° + \theta}{2} tan\frac{45° - \theta}{2}$$

$$= tan\frac{45° + \theta}{2} tan\frac{45° - \theta}{2}$$

Formula

(i) cos(A + B) + cos (A – B) = 2cosA cosB

(ii) cos(A - B) - cos (A + B) = 2sinA sinB

$$= \frac{sin\frac{45° + \theta}{2}\ sin\frac{45° - \theta}{2}}{cos\frac{45° + \theta}{2} cos\frac{45° - \theta}{2}}$$

$$= \frac{2sin\frac{45° + \theta}{2}\ sin\frac{45° - \theta}{2}}{2cos\frac{45° + \theta}{2} cos\frac{45° - \theta}{2}}$$

$$= \frac{\cos\left(\frac{45° + \theta}{2} - \frac{45° - \theta}{2}\right) - cos\left(\frac{45° + \theta}{2} + \frac{45° - \theta}{2}\right)}{\cos\left(\frac{45° + \theta}{2} + \frac{45° - \theta}{2}\right) + cos\left(\frac{45° + \theta}{2} - \frac{45° - \theta}{2}\right)}$$

$$= \frac{\cos\left(\frac{45° + \theta - 45° + \theta}{2}\right) - cos\left(\frac{45° + \theta - 45° + \theta}{2}\right)}{\cos\left(\frac{45° + \theta + 45° - \theta}{2}\right) + cos\left(\frac{45° + \theta - 45° + \theta}{2}\right)}$$

$$= \frac{\cos\left(\frac{2\theta}{2}\right) - cos\left(\frac{90°}{2}\right)}{\cos\left(\frac{90°}{2}\right) + cos\left(\frac{2\theta}{2}\right)}$$

$$= \frac{\cos\theta - cos45°}{\cos 45° + cos\theta}$$

$$= \frac{\cos\theta - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} + cos\theta}$$

$$= \frac{\frac{\sqrt{2}cos\theta \ - 1}{\sqrt{2}}}{\frac{\sqrt{2}cos\theta + 1}{\sqrt{2}}}$$

$$= \frac{\sqrt{2}cos\theta \ - 1}{\sqrt{2}cos\theta + 1} \ Proved$$