

Lecture 25

- Resistance and Ohm's law
- chapter 32

Resistance and Ohm's law

- Current created by E (requires ΔV) \Rightarrow ΔV related to I

- E constant by current conservation

$$E = \frac{\Delta V}{\Delta s} = \frac{\Delta V}{L}; I = JA; J = \sigma E \Rightarrow$$
$$I = \frac{A}{\rho L} \Delta V; R \text{ (resistance)} = \frac{\rho L}{A}$$

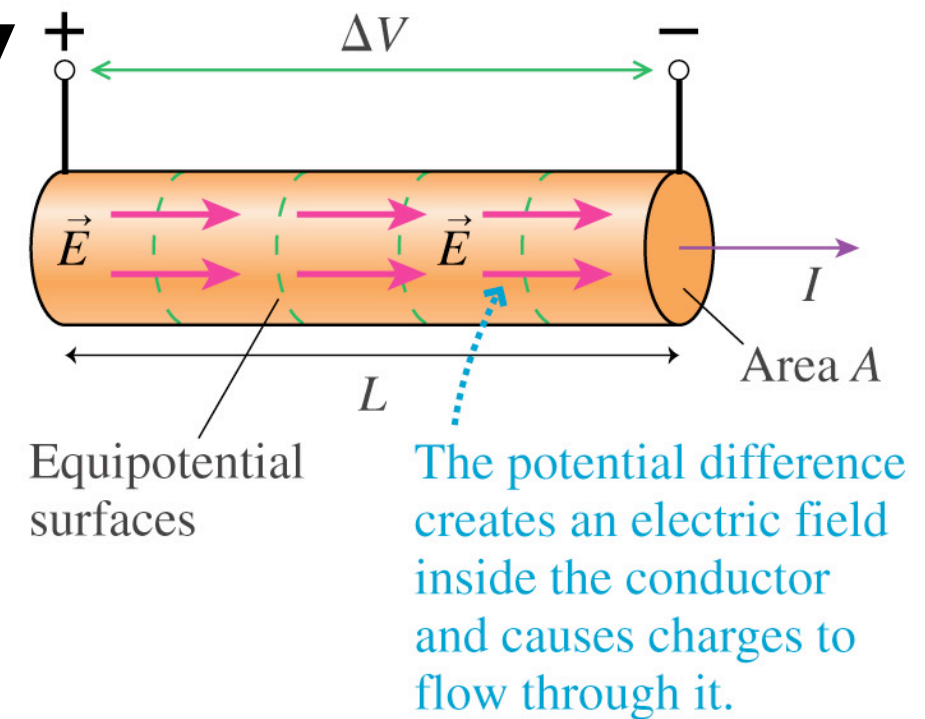
$$I = \frac{\Delta V}{R} \quad (\text{Ohm's law})$$

- R property of specific wire (depends on material and L, A):

Unit of R : $1 \text{ ohm} = 1 \Omega \equiv 1 \text{ V/A}$

(Resistivity: property of material only)

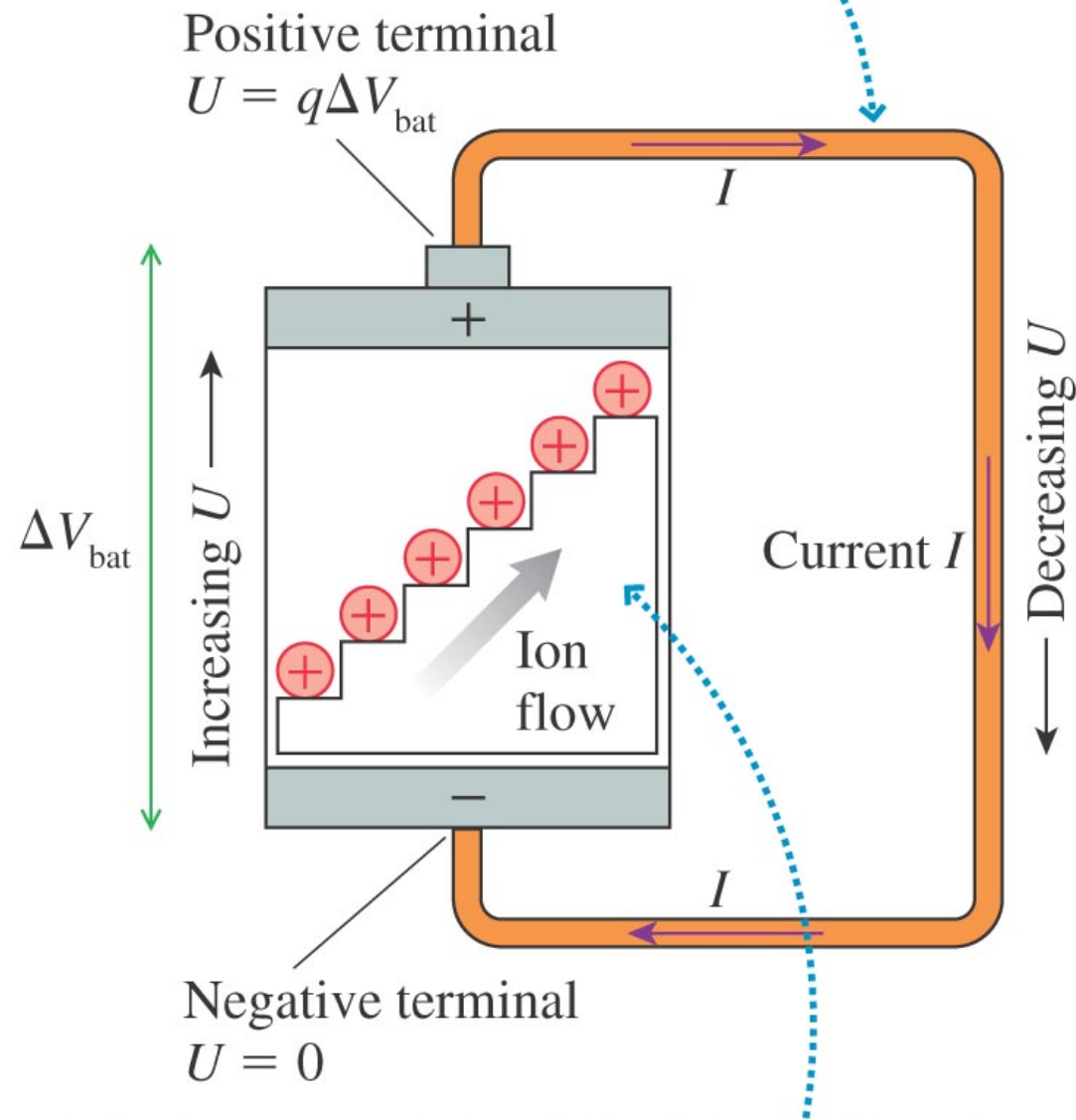
- resistors: circuit elements with resistance larger than wires used to limit current



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Batteries and current

The charge “falls downhill” through the wire, but a current can be sustained because of the charge escalator.

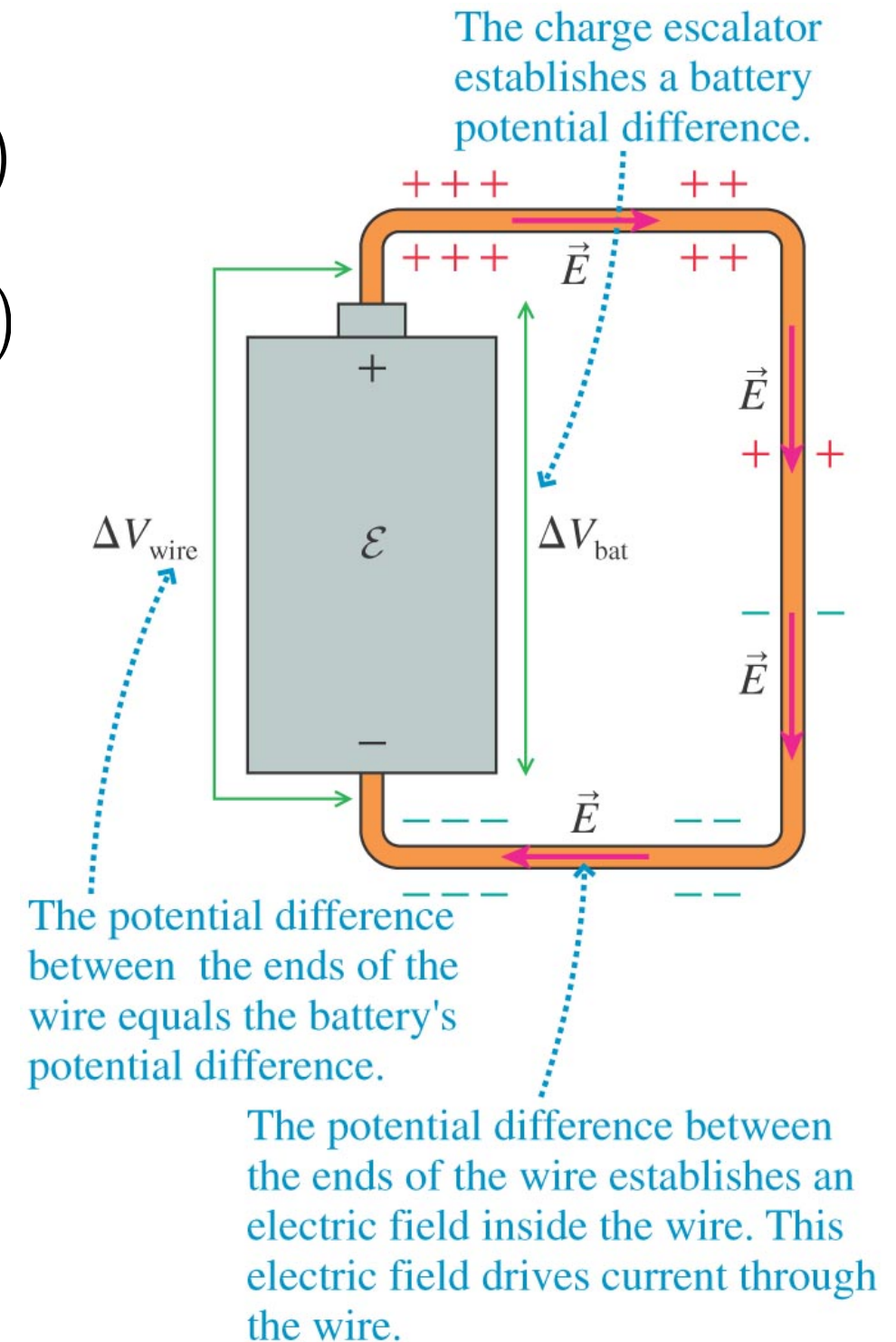


The charge escalator “lifts” charge from the negative side to the positive side. Charge q gains energy $\Delta U = q\Delta V_{\text{bat}}$.

- so far, current from discharge of capacitor: transient (stops when excess charge removed)
- battery: sustained current due energy from chemical reactions used to “lift” charge...falls “downhill” in wire (warms it)

Cause and Effect

- Battery source of ΔV_{bat} ($= \mathcal{E}$ for ideal)
 $\Delta V_{wire} = \Delta V_{bat}$ (independent of path)
- ΔV_{wire} causes $E_{wire} = \frac{\Delta V_{wire}}{L}$ (surface charges)
- E results in current: $I = JA = \sigma AE$
- Current determined by **both** battery and wire's R



Ohmic and Nonohmic materials; Ideal Wire Model

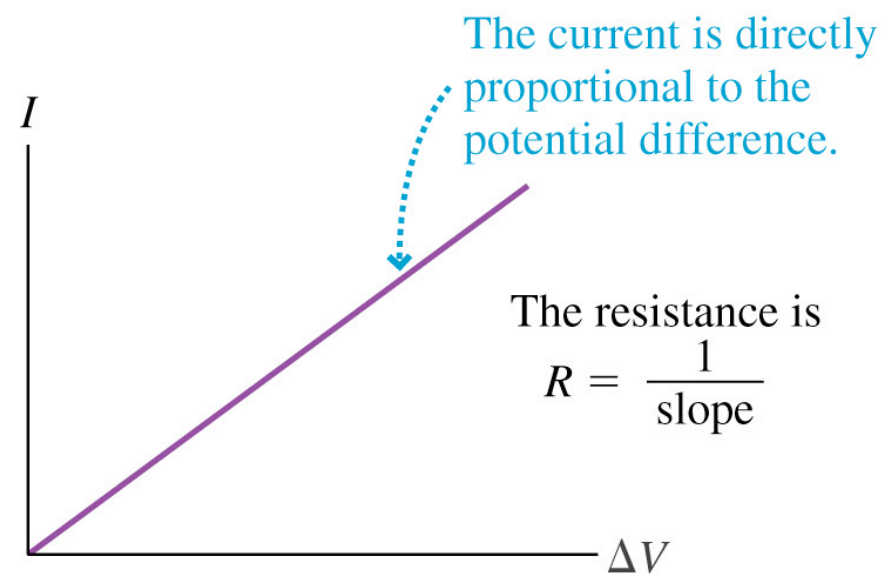
- ideal wires: $R = 0 \Rightarrow \Delta V = 0$ even $I \neq 0$

- resistors: 10 to $10^6 \Omega$

- ideal insulators: $R = \infty \Rightarrow$

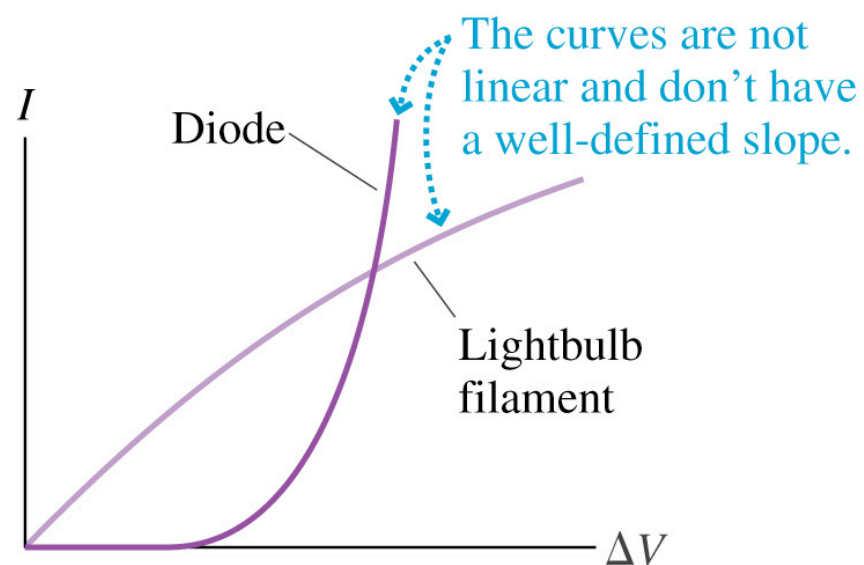
$$I = 0 \text{ even if } \Delta V \neq 0$$

(a) Ohmic material

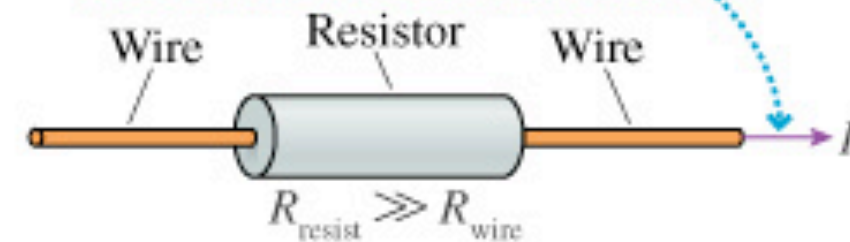


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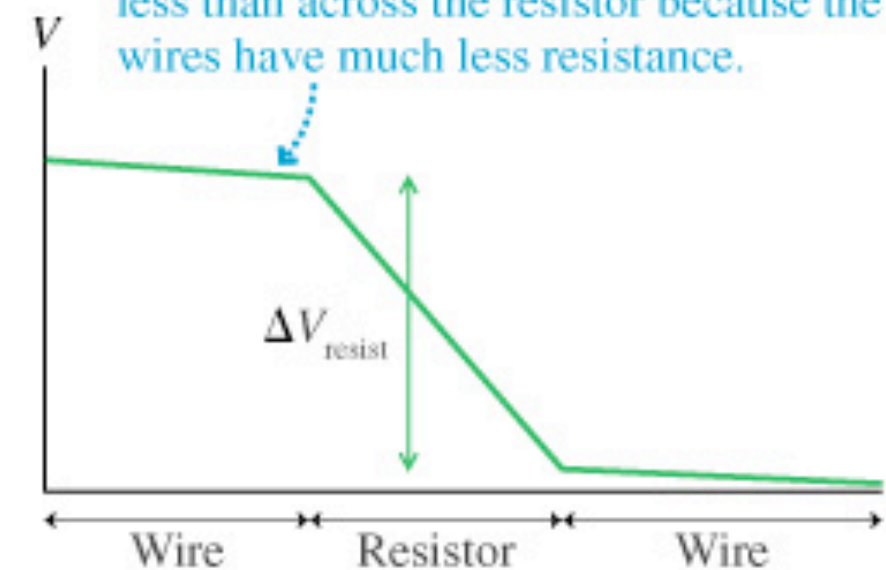
(b) Nonohmic materials



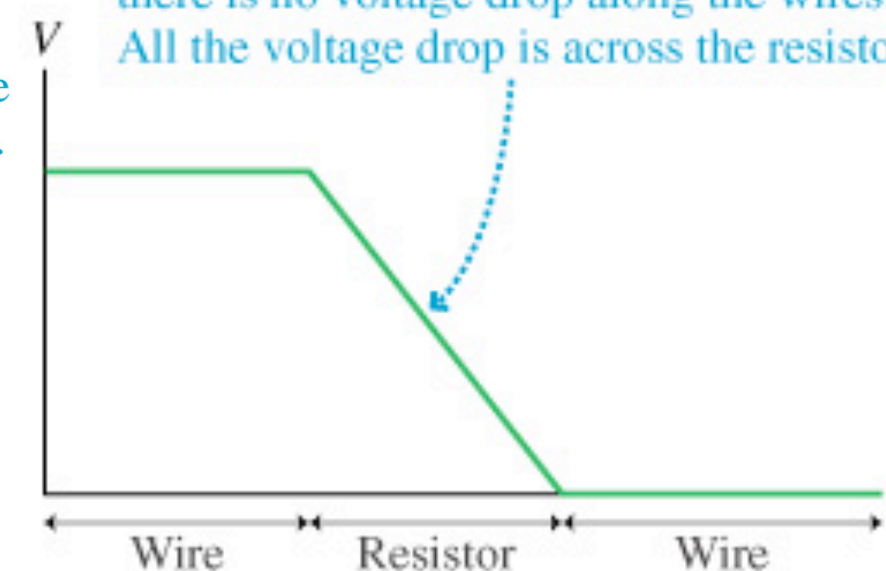
(a) The current is constant along the wire-resistor-wire combination.



(b) The voltage drop along the wires is much less than across the resistor because the wires have much less resistance.



(c) In the ideal wire model, with $R_{\text{wire}} = 0 \Omega$, there is no voltage drop along the wires. All the voltage drop is across the resistor.

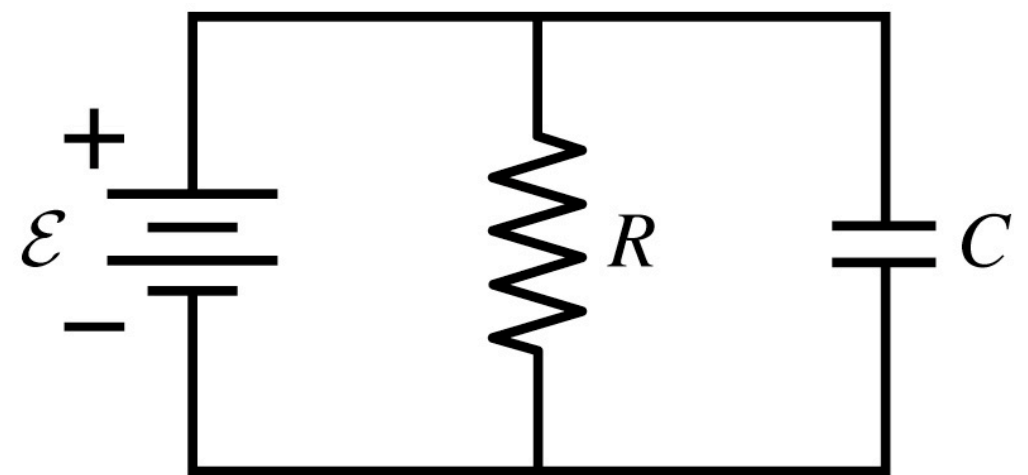
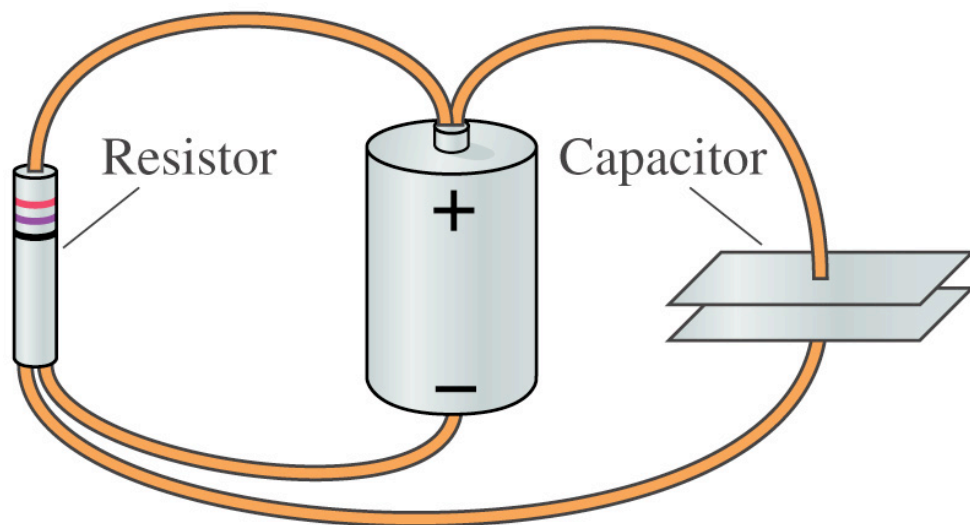
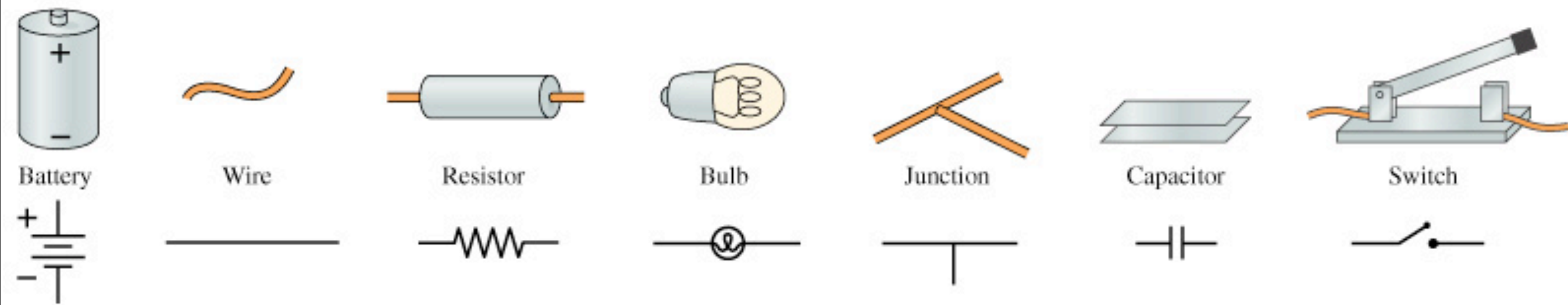


Chapter 3 I (Fundamentals of Circuits)

- understand fundamental principles of electric circuits; direct current (DC): battery's potential difference, currents constant
 - Kirchhoff's Laws and Basic Circuit
 - Energy and Power
 - Resistors in Series
 - Real batteries
 - Resistors in Parallel

Circuit Elements and Diagrams

- circuit diagram: logical picture of connections (replace pictures of circuit elements by symbols)



Kirchhoff's Laws

- circuit analysis: finding potential difference across and current in each component
- junction law (charge conservation)

$$\sum I_{in} = \sum I_{out}$$

- loop law (energy conservation)

$$\Delta V_{loop} = \sum_i (\Delta V)_i = 0$$

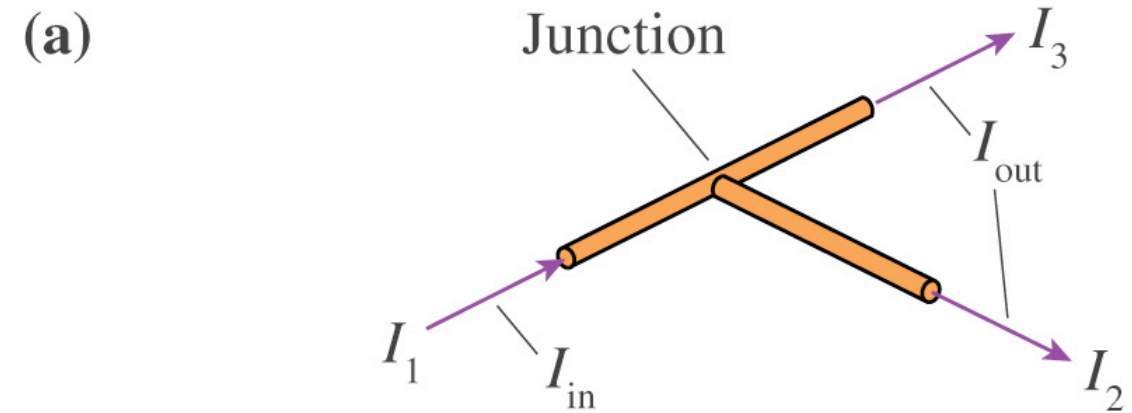
- strategy:

assign current direction

travel around loop in direction of current

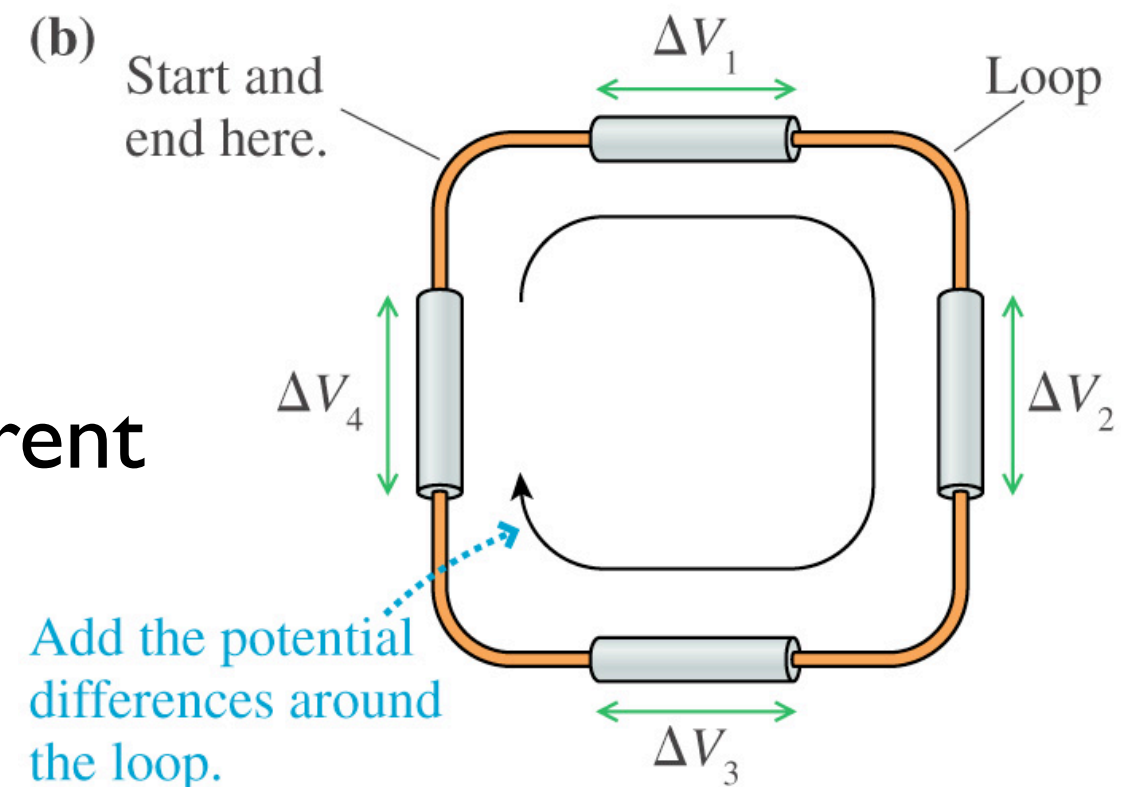
$$V_{bat} = \pm \mathcal{E}; V_R = -IR$$

apply loop law



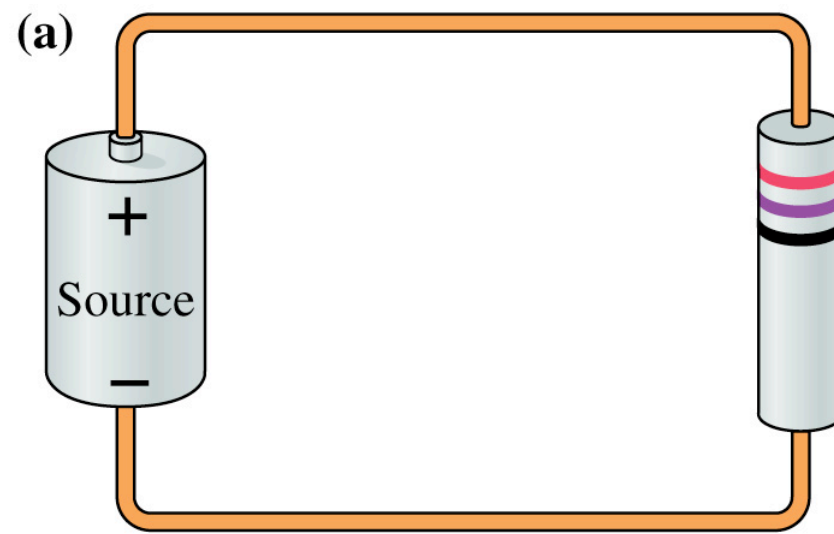
Junction law: $I_1 = I_2 + I_3$

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Loop law: $\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$

Basic Circuit



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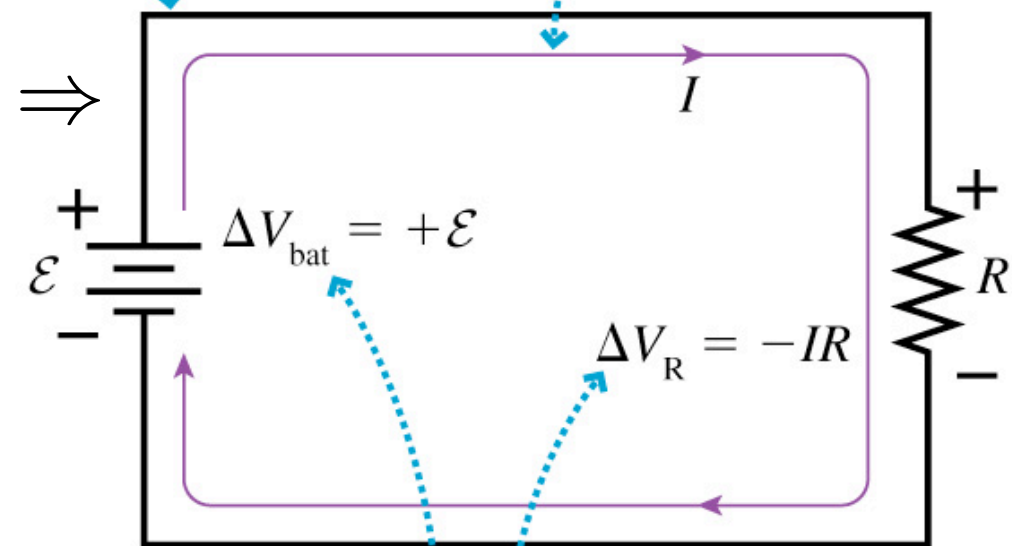


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- junction law not needed
- ideal wires: no potential difference
- loop law: $\Delta V_{loop} = \Delta V_{bat} + \Delta V_R = 0$
 $\Delta V_{bat} = +\mathcal{E};$
 $\Delta V_R = V_{downstream} - V_{upstream} = -IR \Rightarrow$
 $\mathcal{E} - IR = 0; I = \frac{\mathcal{E}}{R}; \Delta V_R = -IR = -\mathcal{E}$

1 Draw circuit diagram.

2 The orientation of the battery indicates a clockwise current, so assign a clockwise direction to I .



3 Determine ΔV for each circuit element.

A more complex circuit

- charge can flow “backwards” thru’ battery: choose cw direction for current (if solution negative, current is really ccw)

- loop law: $\Delta V_{bat\ 1} + \Delta V_{R1} + \Delta V_{bat\ 2} + \Delta V_{R2} = 0$

$$\Delta V_{bat\ 1} = +\mathcal{E}_1; \Delta V_{bat\ 2} = -\mathcal{E}_2;$$

$$\Delta V_{R1} = -IR_1; \Delta V_{R2} = -IR_2 \Rightarrow$$

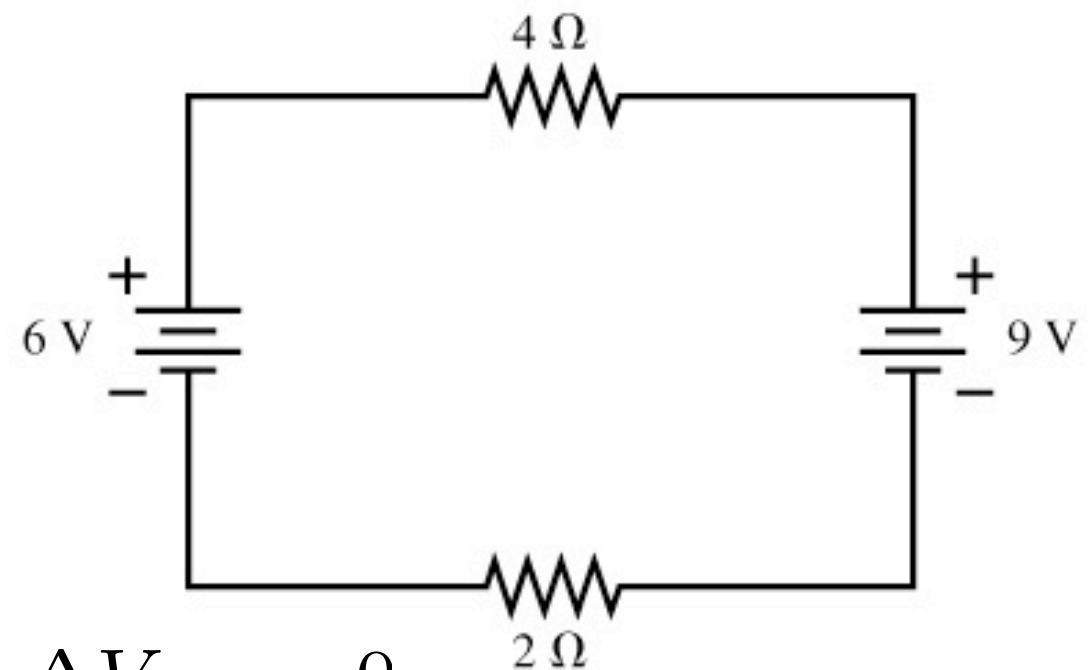
$$\mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0 \Rightarrow$$

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{6\text{ V} - 9\text{ V}}{4\Omega + 2\Omega} = -0.5\text{ A}...$$

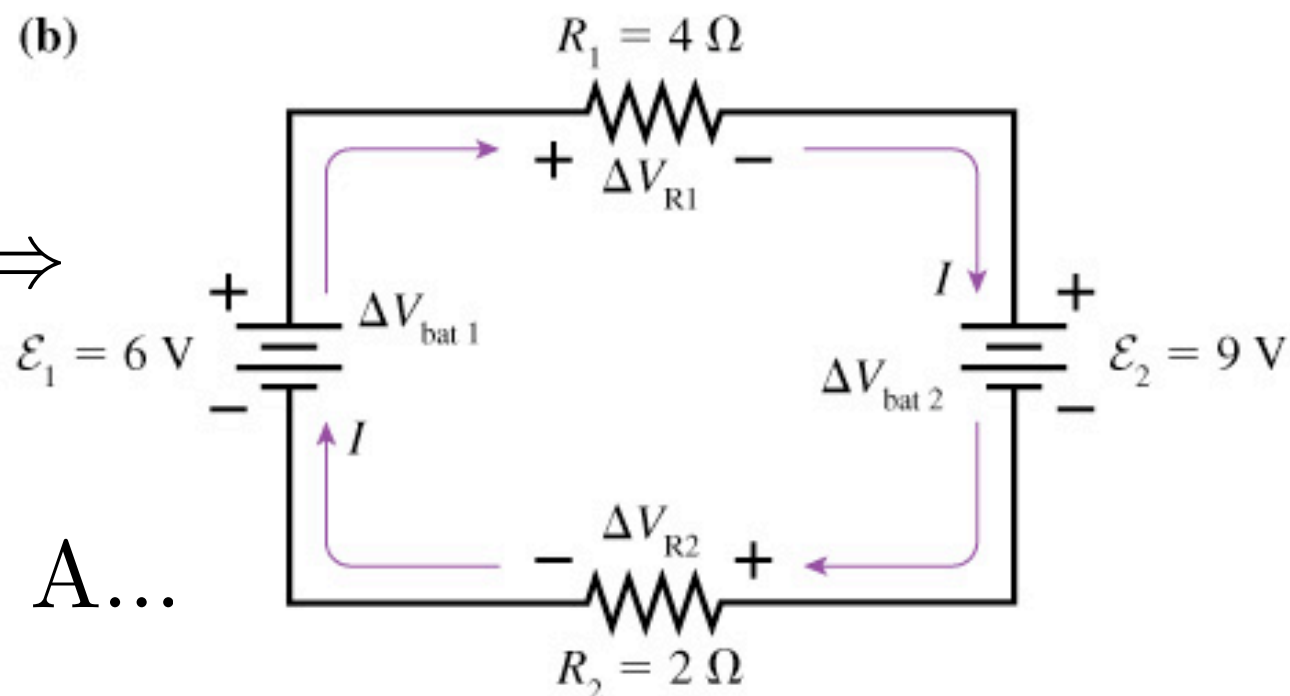
(expected: 9 V battery “dictates” direction...)

$$\Delta V_{R1} = -IR_1 = +2.0\text{ V}...$$

(a)



(b)



Energy and Power

- In battery: chemical energy potential...of charges: $\Delta U = q\Delta V_{bat} = q\mathcal{E}$
- power (rate at which energy supplied to charges): $P_{bat} = \frac{dU}{dt} = \frac{dq}{dt}\mathcal{E}$

$$P_{bat} = I\mathcal{E} \quad (\text{power delivered by an emf})$$

- In resistor: work done on charges $qEd \rightarrow$ kinetic (accelerate) between collisions \rightarrow thermal energy of lattice after collisions

After many collisions over length L of resistor:

$$\Delta E_{th} = qEL = q\Delta V_R$$

rate at which energy is transferred from current to resistor:

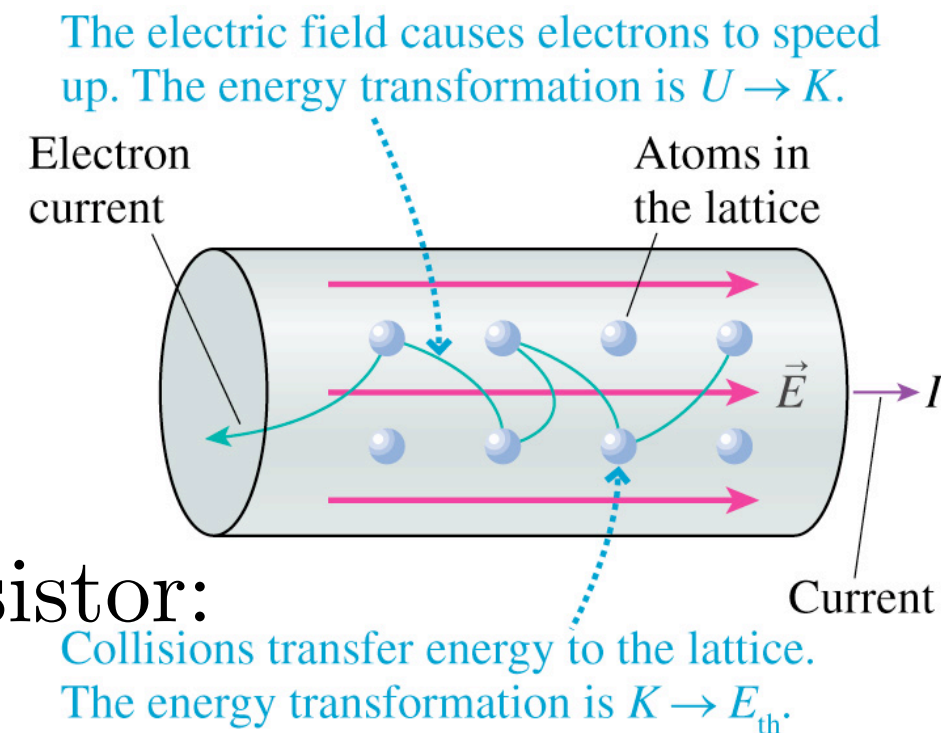
$$P_R = \frac{dE_{th}}{dt} = \frac{dq}{dt}\Delta V_R = I\Delta V_R$$

For basic circuit: $P_R = P_{bat}$ (energy conservation)

Using Ohm's law: $P_R = I\Delta V_R = I^2R = \frac{(\Delta V_R)^2}{R}$ (power dissipated by a resistor)

$$E_{chem} \rightarrow U \rightarrow K \rightarrow E_{th} \rightarrow \text{light}...$$

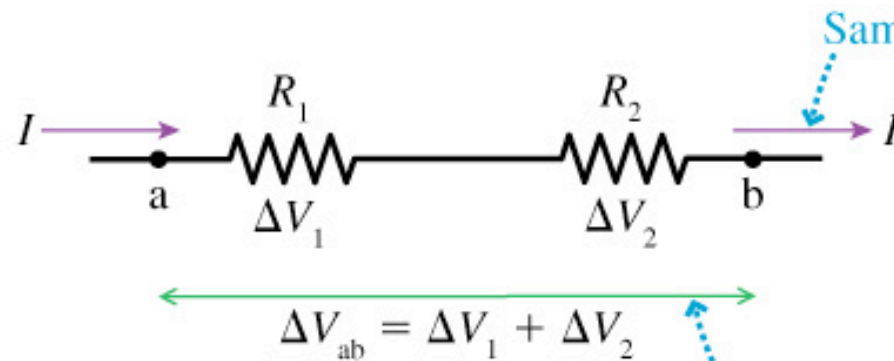
- Common units: P_R kW in Δt hours $\rightarrow P_R\Delta t$ kilowatt hours



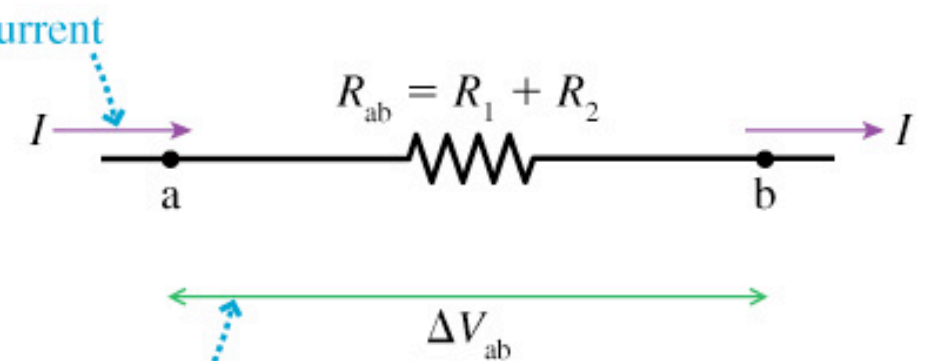
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Resistors in Series

(a) Two resistors in series



(b) An equivalent resistor



Same current

Same potential difference

- current same in each resistor: $\Delta V_1 = IR_1$; $\Delta V_2 = IR_2 \Rightarrow \Delta V_{ab} = I(R_1 + R_2)$
- equivalent resistor: $R_{ab} = \frac{\Delta V_{ab}}{I} = \frac{I(R_1 + R_2)}{I} = R_1 + R_2$

$$R_{eq} = R_1 + R_2 + \cdots + R_N \quad (\text{series resistors})$$

Ammeters

- measures current in circuit element (placed in series):

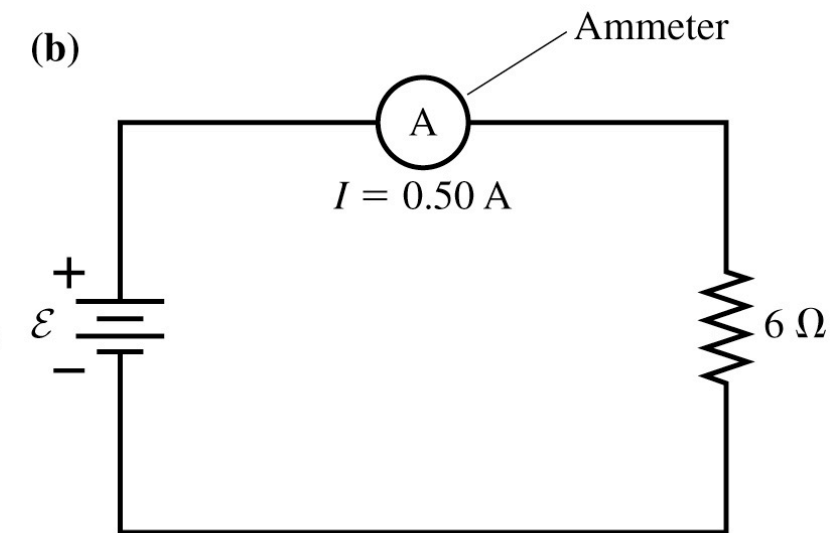
$$R_{eq} = R_{load} + R_{ammeter}$$

ideal, $R_{ammeter} = 0 \Rightarrow$ current not changed

(a)



(b)



Real Batteries

- terminal (user) voltage

$$\Delta V_{bat} = E - I r$$

- for resistor connected:

$$I = \frac{E}{R_{eq}} = \frac{E}{R + r}$$

$$\Delta V_R = I R = \frac{R}{R + r} E$$

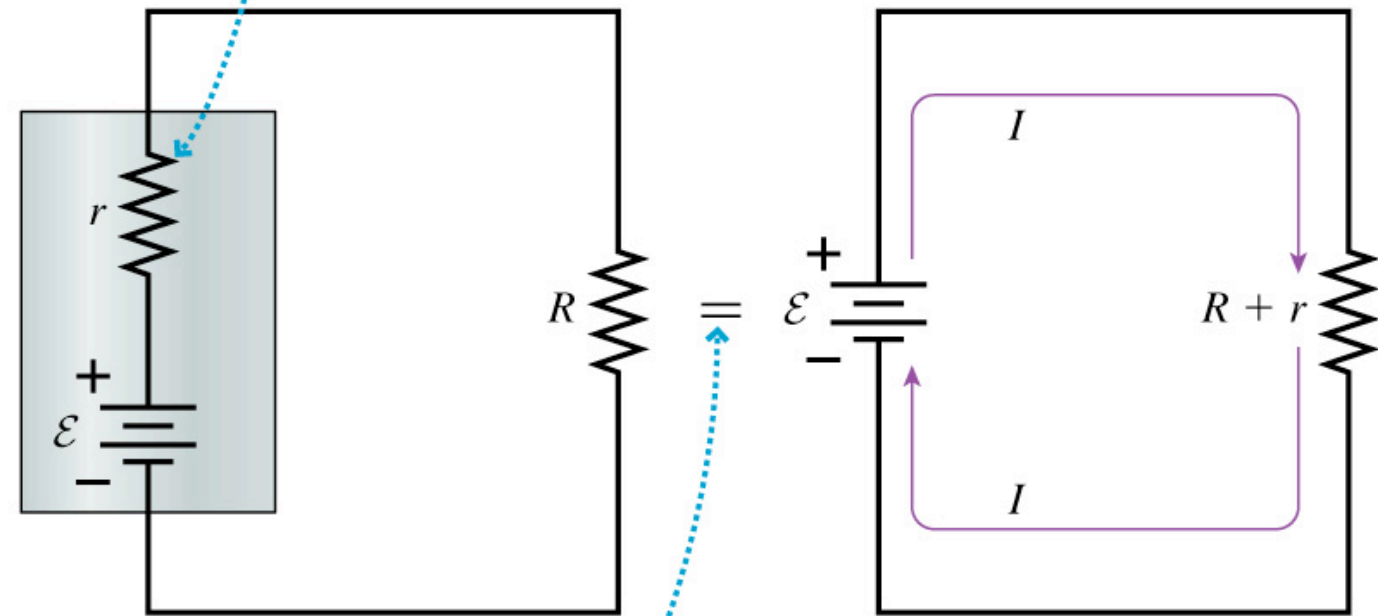
$$\Delta V_{bat} = E - I r;$$

$$\Delta V_R = \Delta V_{bat} = E$$

- replace high by zero resistance $\rightarrow I_{short} = \frac{E}{r}$
(∞ for ideal, $r = 0$)

- maximum possible current battery can produce

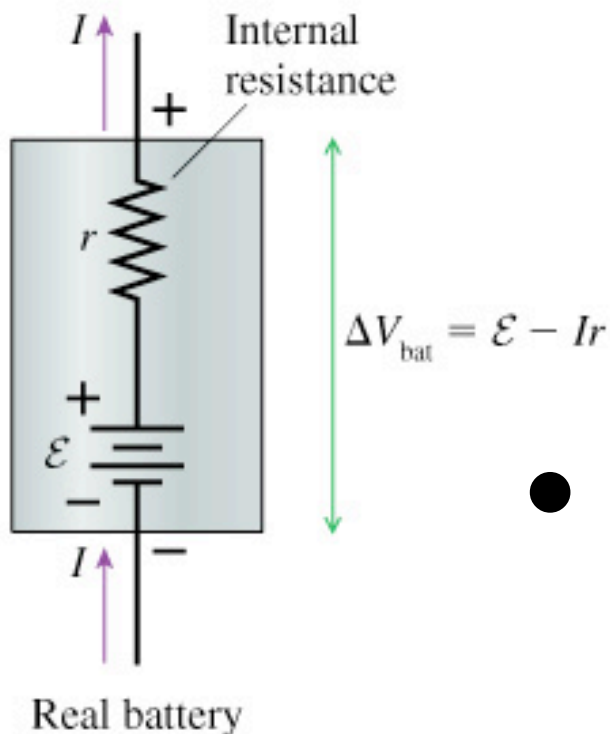
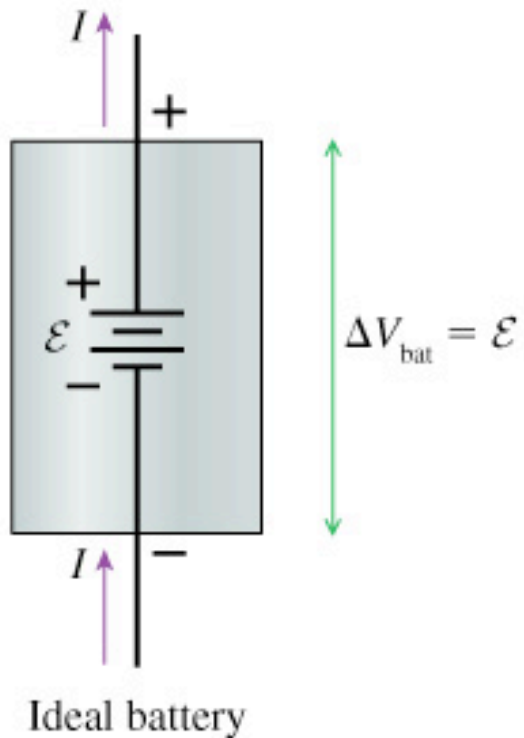
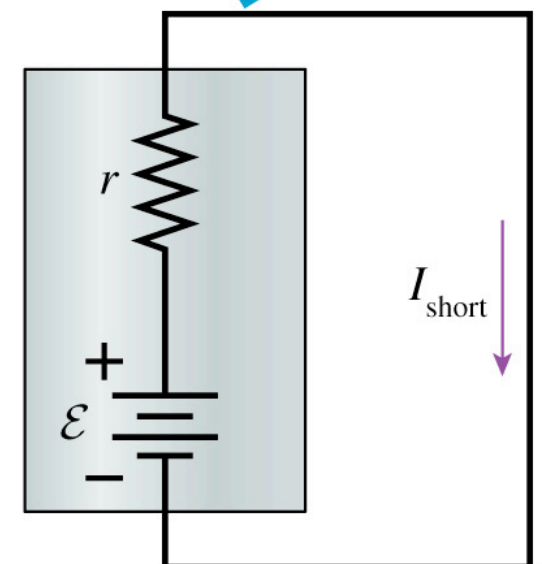
Although physically separated, the internal resistance r is electrically in series with R .



This means the two circuits are equivalent.

Short Circuit

This wire is shorting out the battery.



Parallel Resistors

- potential differences same:

$$\Delta V_1 = \Delta V_2 = \dots = V_{cd}$$

- Kirchhoff's junction law: $I = I_1 + I_2$

- Ohm's law:

$$I = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} = \Delta V_{cd} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

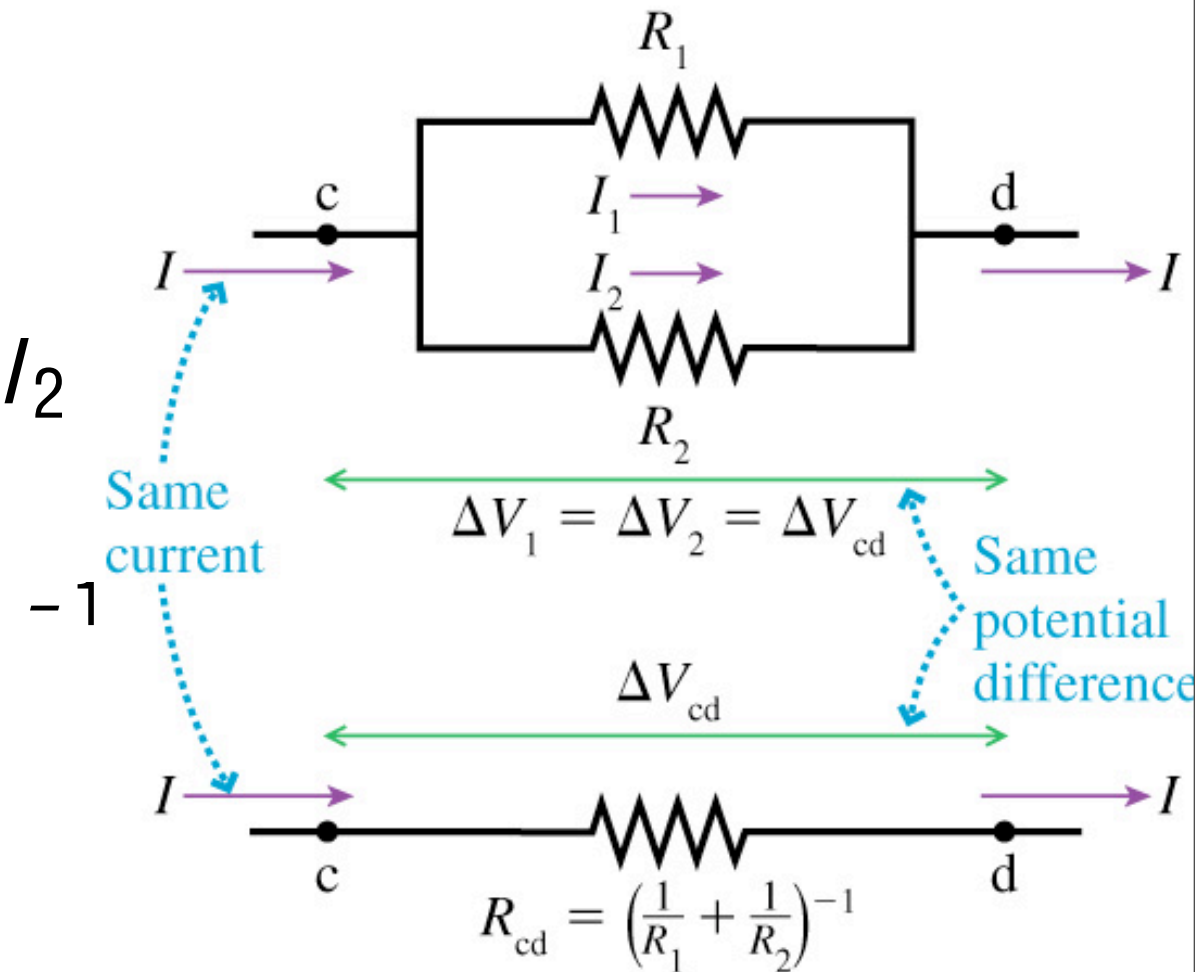
- Replace by equivalent resistance:

$$R_{cd} = \frac{\Delta V_{cd}}{I} = \left[\frac{1}{R_1} + \frac{1}{R_2} \right]^{-1}$$

$$R_{eq} = \left[\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N} \right]^{-1} \quad (\text{parallel resistors})$$

- Identical resistors ($R_1 = R_2$): $R_{series\ eq} = 2R$; $R_{parallel\ eq} = \frac{R}{2}$
- In general, $R_{eq} < R_1$ or R_2 ...in parallel

(a) Two resistors in parallel



(b) An equivalent resistor

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