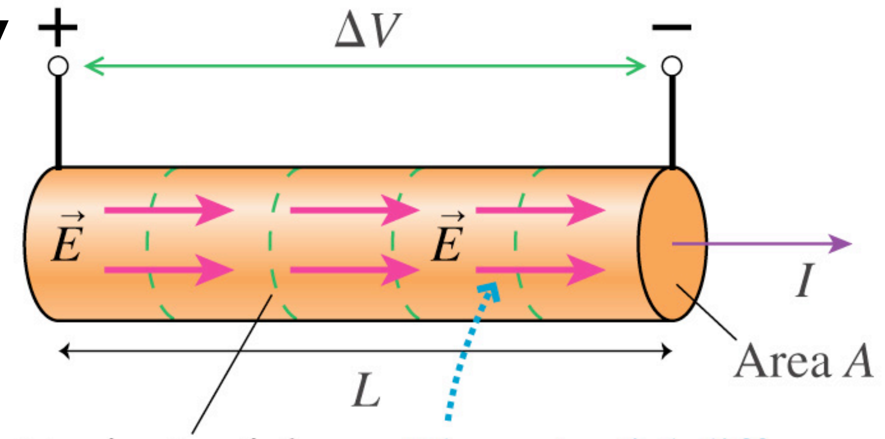
Lecture 25

- Resistance and Ohm’s law
- chapter 32

Resistance and Ohm's law



- Current created by E (requires ΔV)

ΔV related to I

- E constant by current conservation

$$E = \frac{\Delta V}{\Delta s} = \frac{\Delta V}{L};\ I = JA;\ J = \sigma E \Rightarrow$$
$$I = \frac{A}{\rho L}\Delta V;\ R\ (\textbf{resistance}) = \frac{\rho L}{A}$$

$$I = \frac{\Delta V}{R} \qquad \text{(Ohm's law)}$$

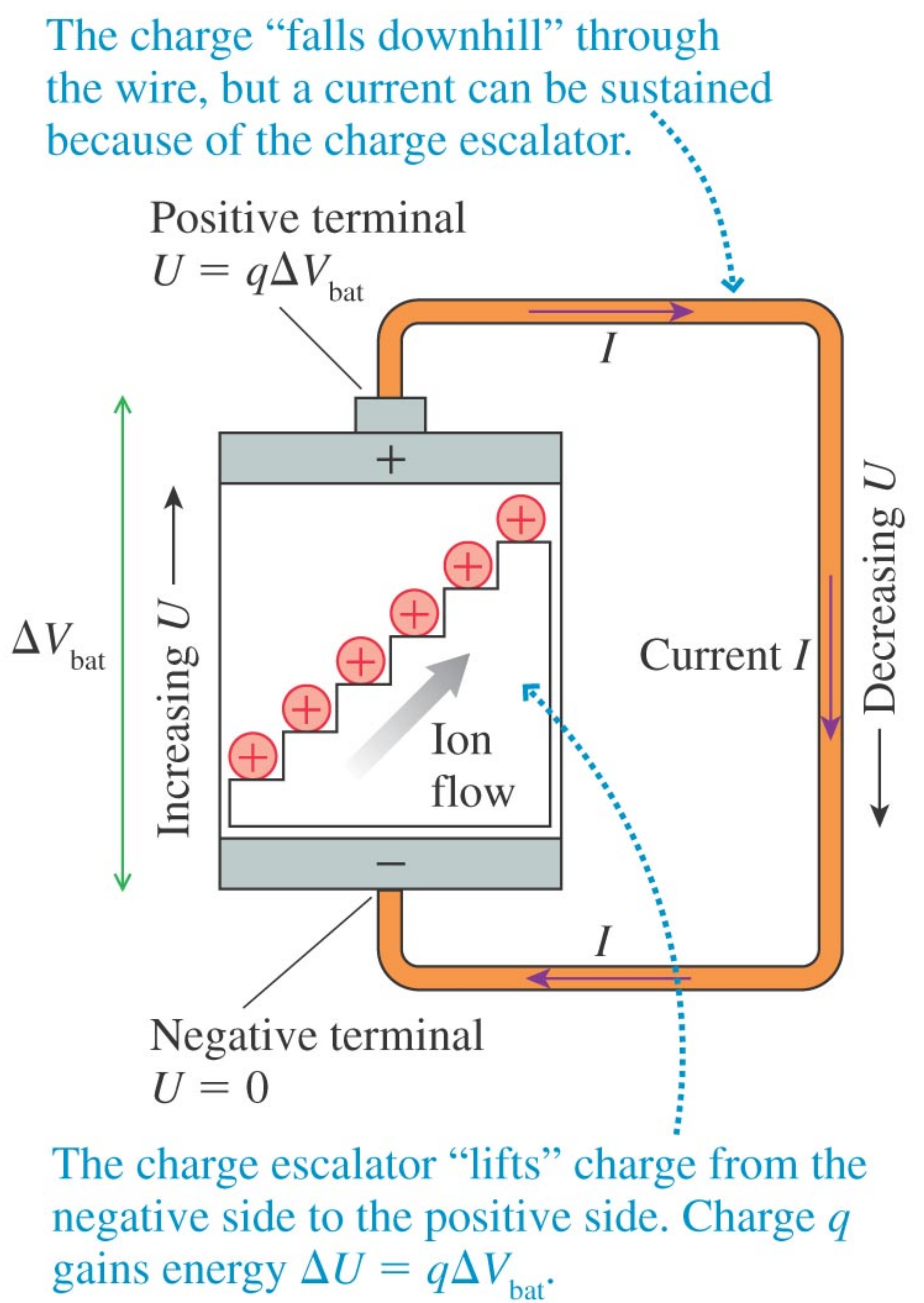
- R property of specific wire (depends on material and L, A):

Unit of R: 1 ohm $= 1\ \Omega \equiv 1$ V/A

(Resistivity: property of material only)

- resistors: circuit elements with resistance larger than wires used to limit current

Batteries and current



- so far, current from discharge of capacitor: transient (stops when excess charge removed)
- battery: sustained current due energy from chemical reactions used to "lift" charge...falls "downhill" in wire (warms it)

Cause and Effect

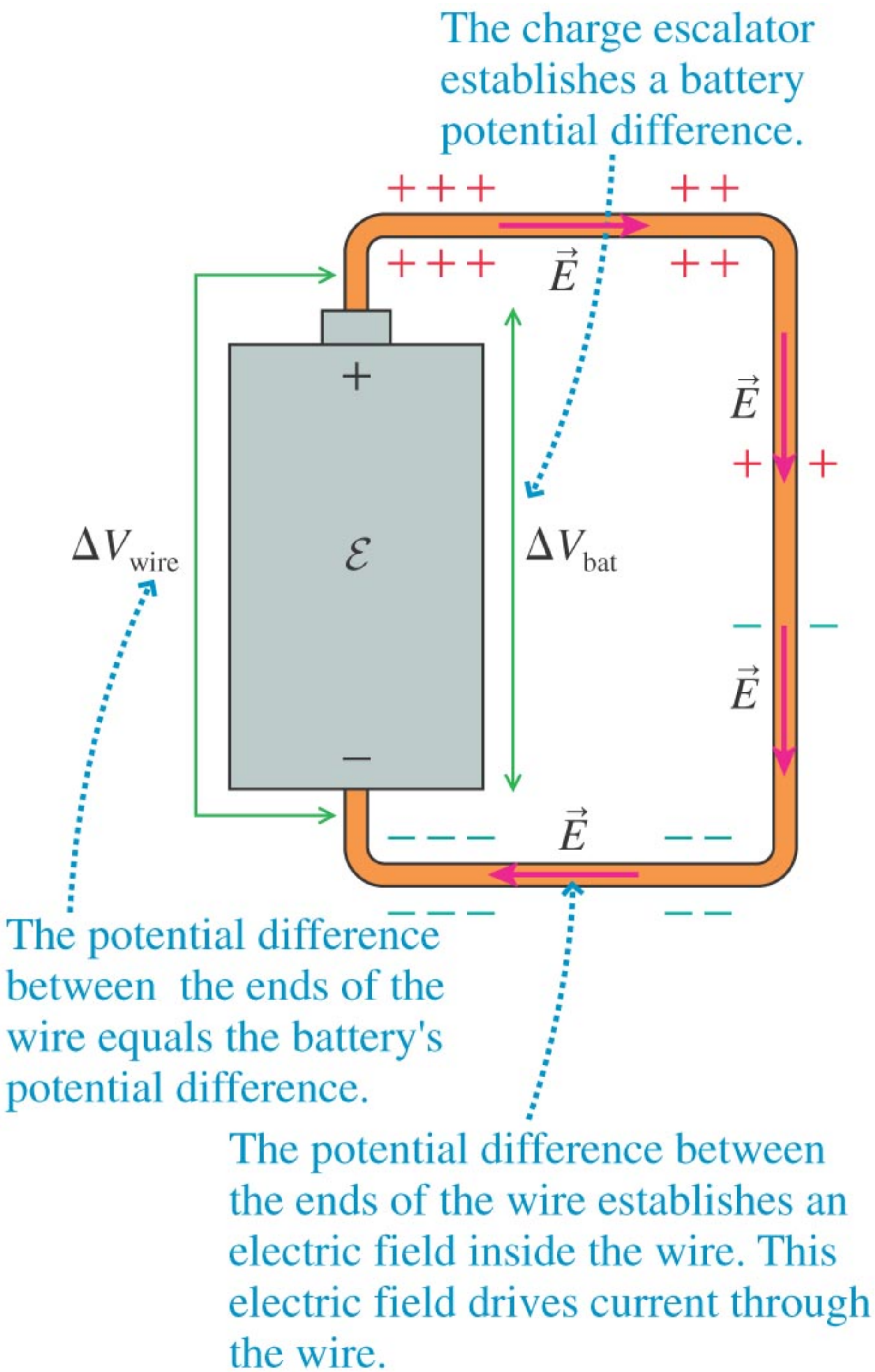
- Battery source of ΔV_{bat} $(= \mathcal{E}$ for ideal)

$$\Delta V_{wire} = \Delta V_{bat} \text{ (independent of path)}$$

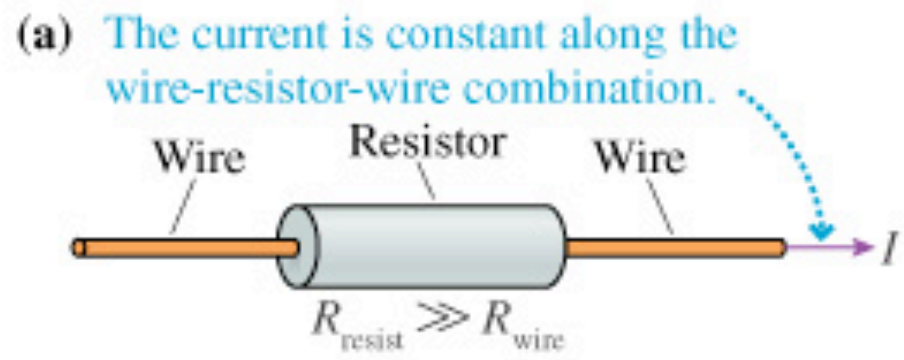
- ΔV_{wire} causes $E_{\text{wire}} = \frac{\Delta V_{\text{wire}}}{L}$ (surface charges)

- E results in current: $I = JA = \sigma AE$

- Current determined by both battery and wire's R



Ohmic and Nonohmic materials; Ideal Wire Model



- ideal wires: $R = 0 \Rightarrow$

  $\Delta V = 0 \text{ even } I \neq 0$

- resistors:

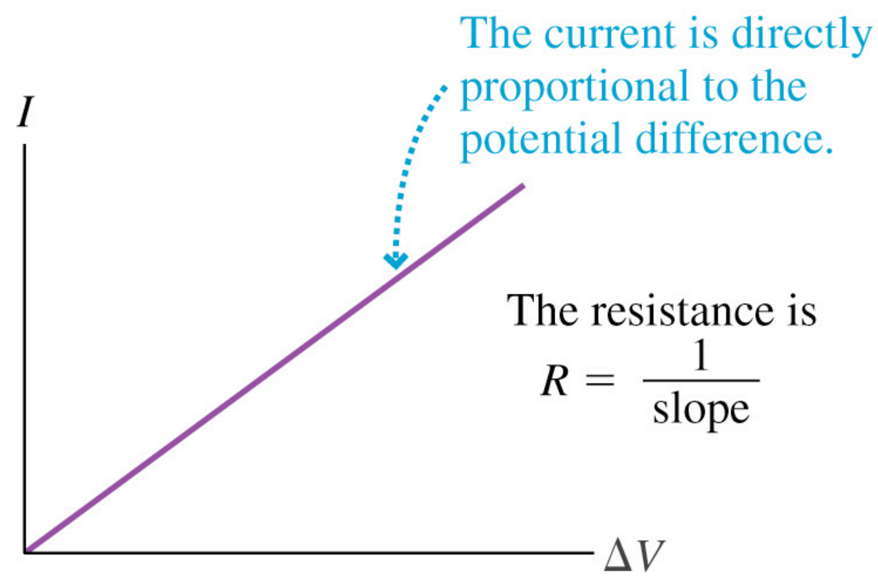
  $10 \text{ to } 10^6 \Omega$

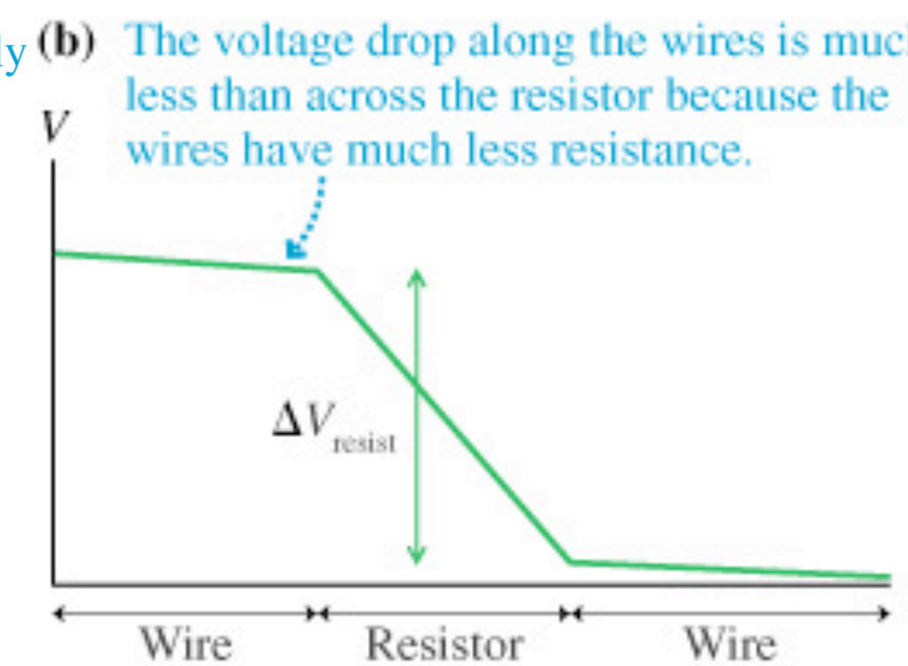
- ideal insulators:

  $R = \infty \Rightarrow$

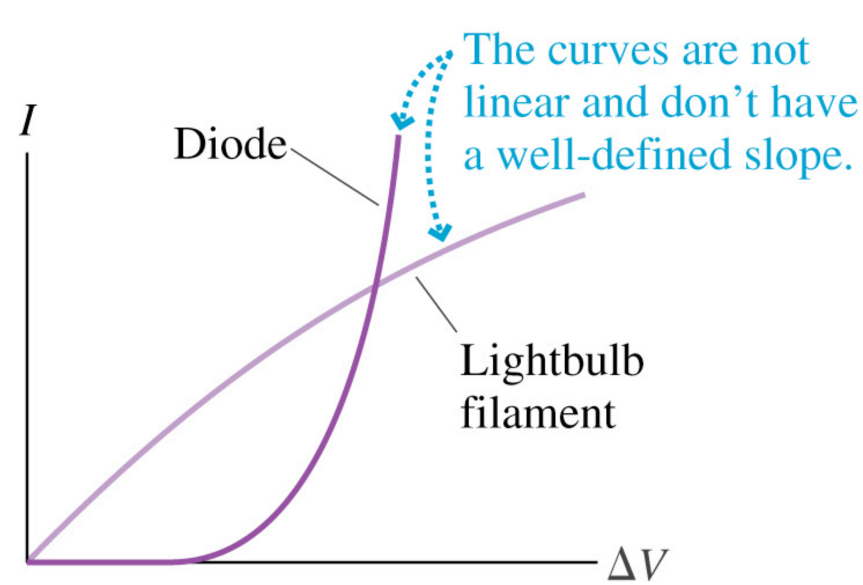
  $I = 0 \text{ even if } \Delta V \neq 0$

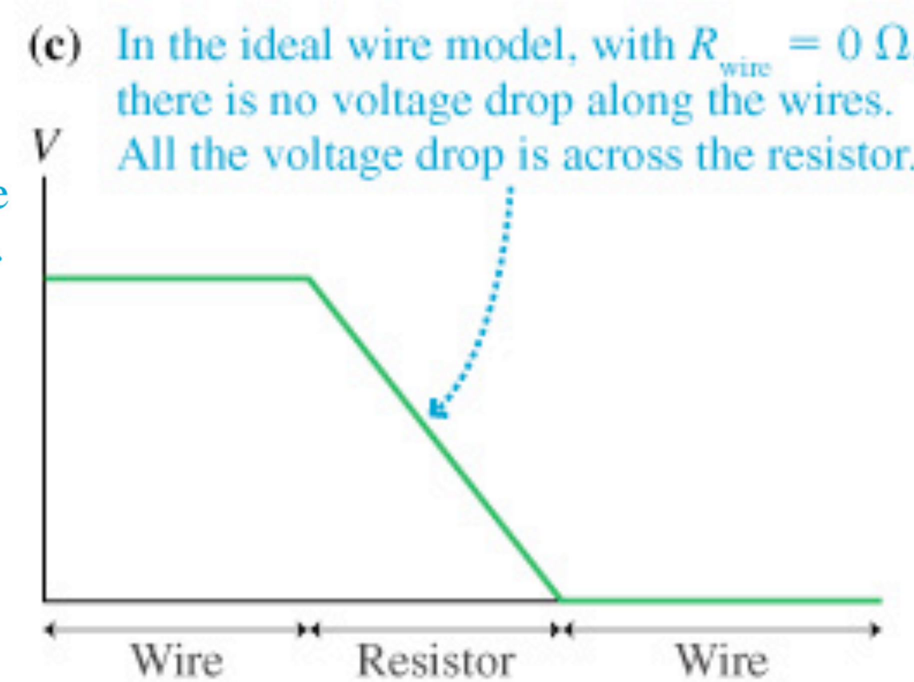
(a) Ohmic material





(b) Nonohmic materials



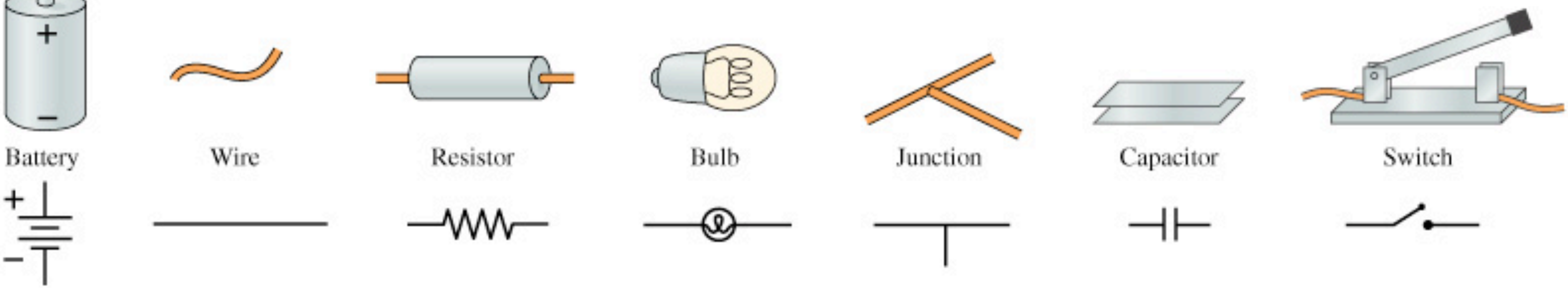


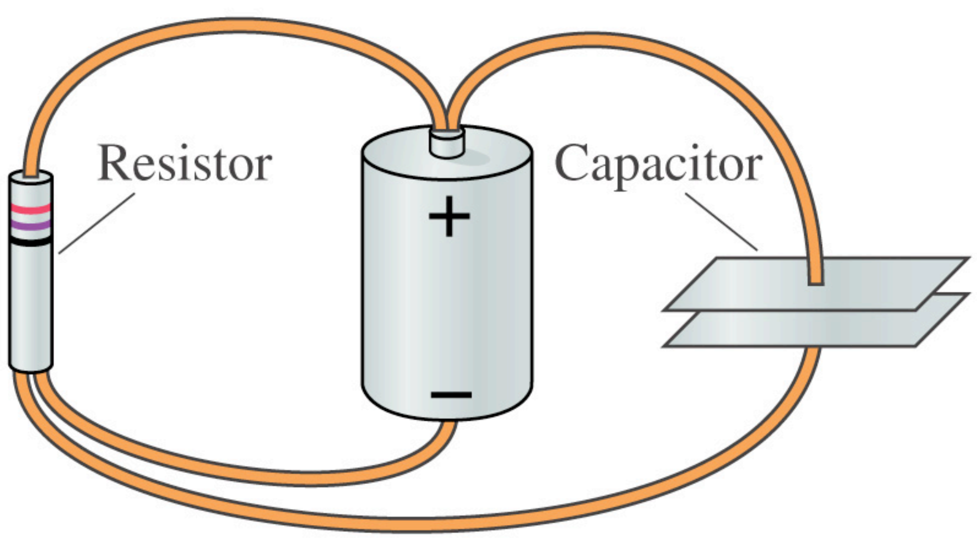
Chapter 31 (Fundamentals of Circuits)

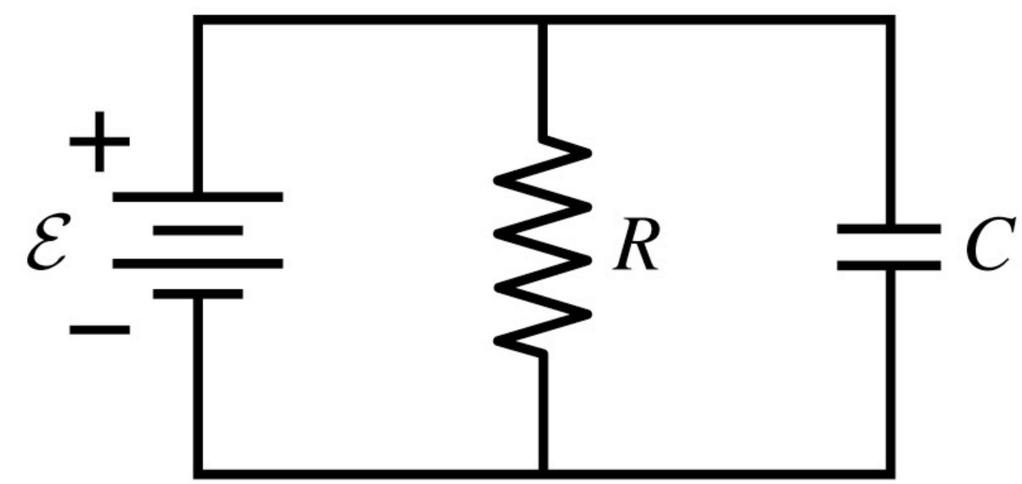
- understand fundamental principles of electric circuits; direct current (DC): battery's potential difference, currents constant
    - Kirchhoff's Laws and Basic Circuit
    - Energy and Power
    - Resistors in Series
    - Real batteries
    - Resistors in Parallel

Circuit Elements and Diagrams

- circuit diagram: logical picture of connections (replace pictures of circuit elements by symbols)







Kirchhoff's Laws

- circuit analysis: finding potential difference across and current in each component
- junction law (charge conservation)

$$\sum I_{in} = \sum I_{out}$$

- loop law (energy conservation)

$$\Delta V_{loop} = \sum_i (\Delta V)_i = 0$$

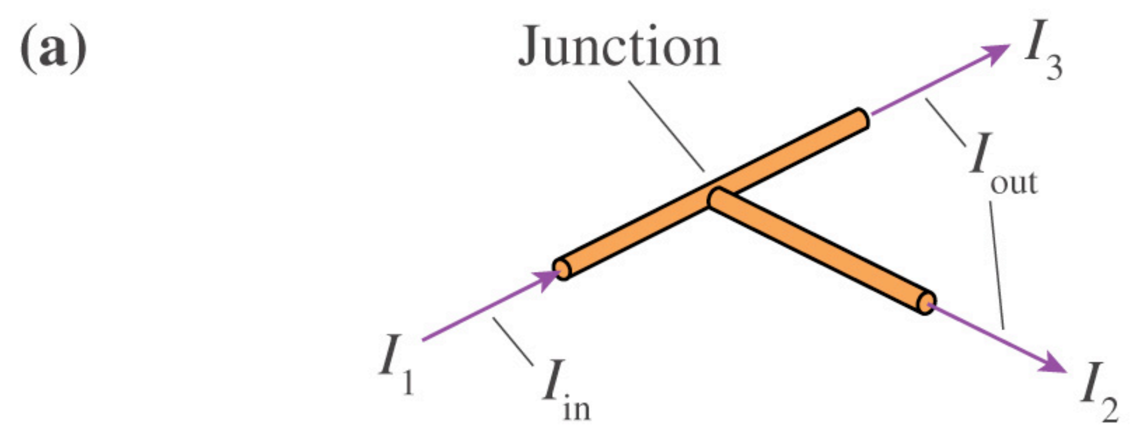
- strategy:

assign current direction

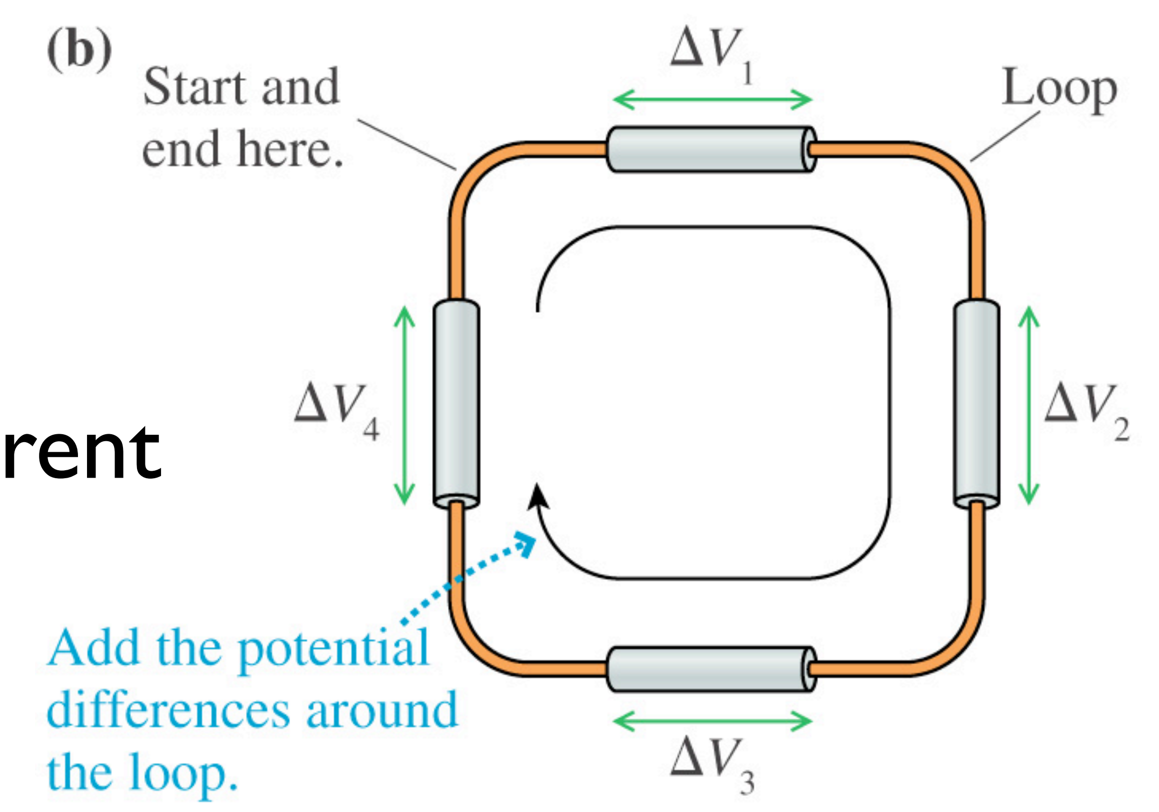
travel around loop in direction of current

$$V_{bat} = \pm \mathcal{E};\, V_R = -IR$$

apply loop law

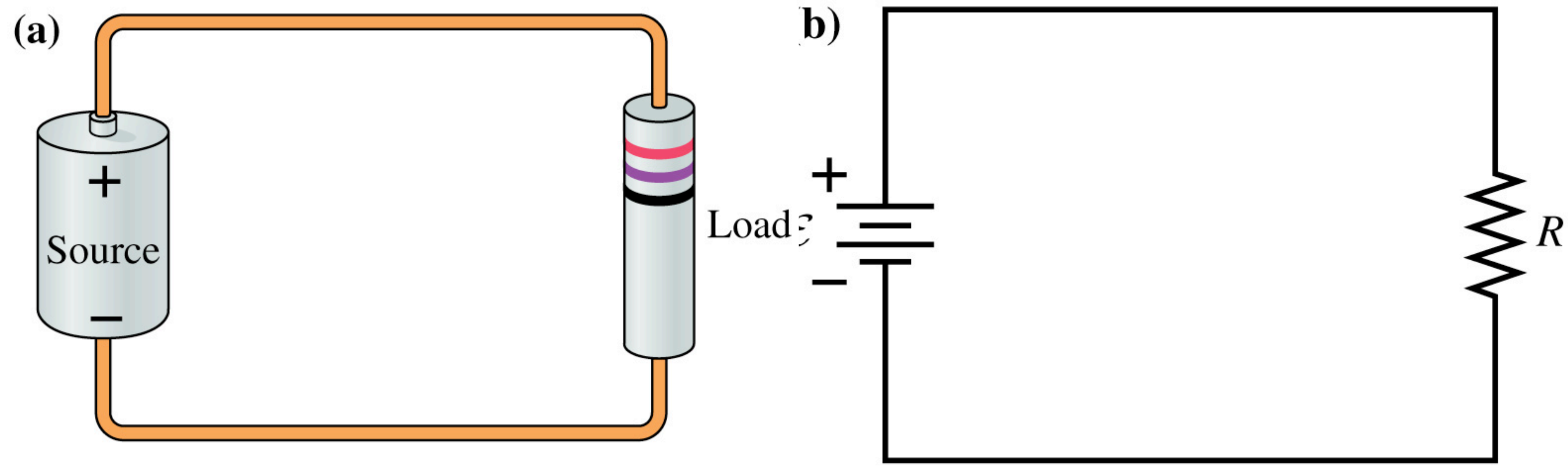


Junction law: $I_1 = I_2 + I_3$



Loop law: $\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$

Basic Circuit

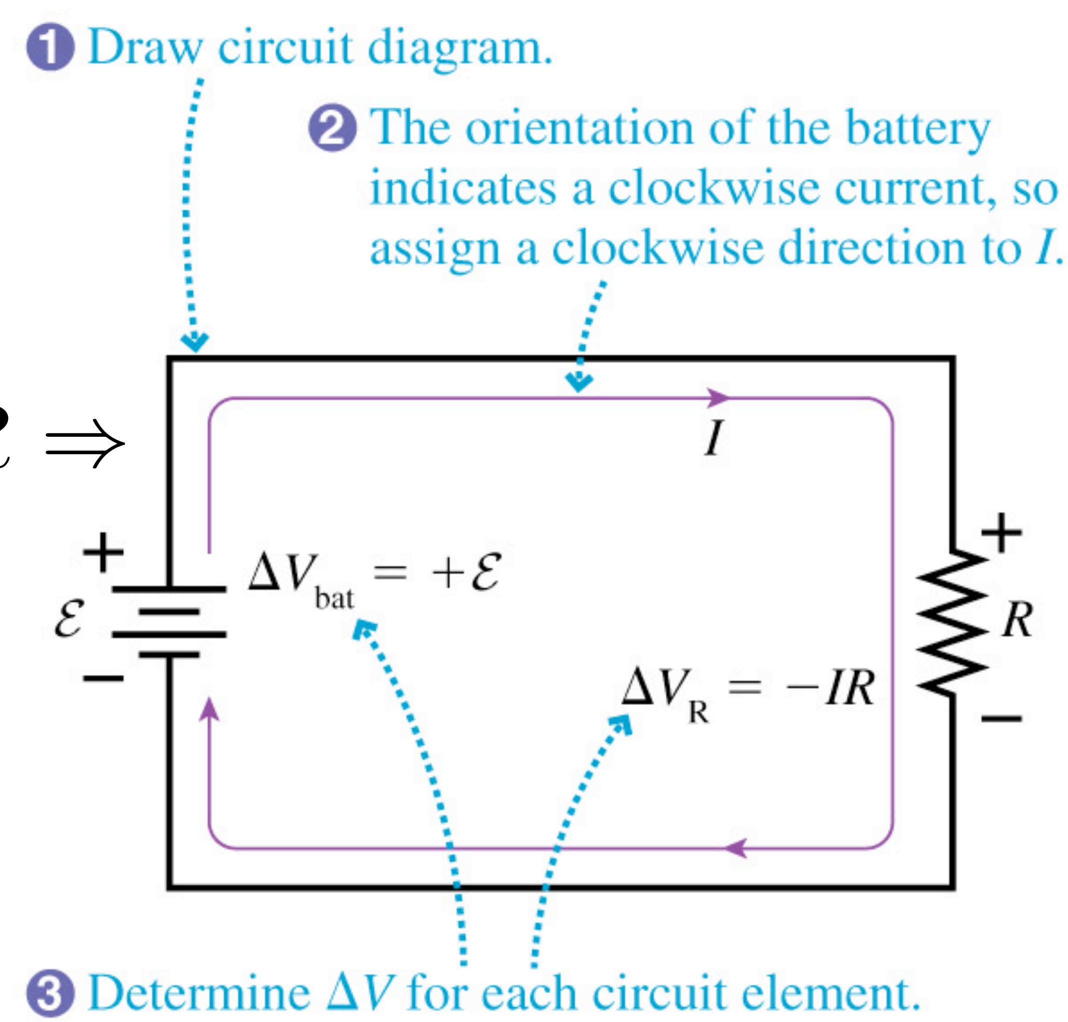


- junction law not needed
- ideal wires: no potential difference
- loop law: $\Delta V_{loop} = \Delta V_{bat} + \Delta V_R = 0$

$\Delta V_{bat} = +\mathcal{E};$

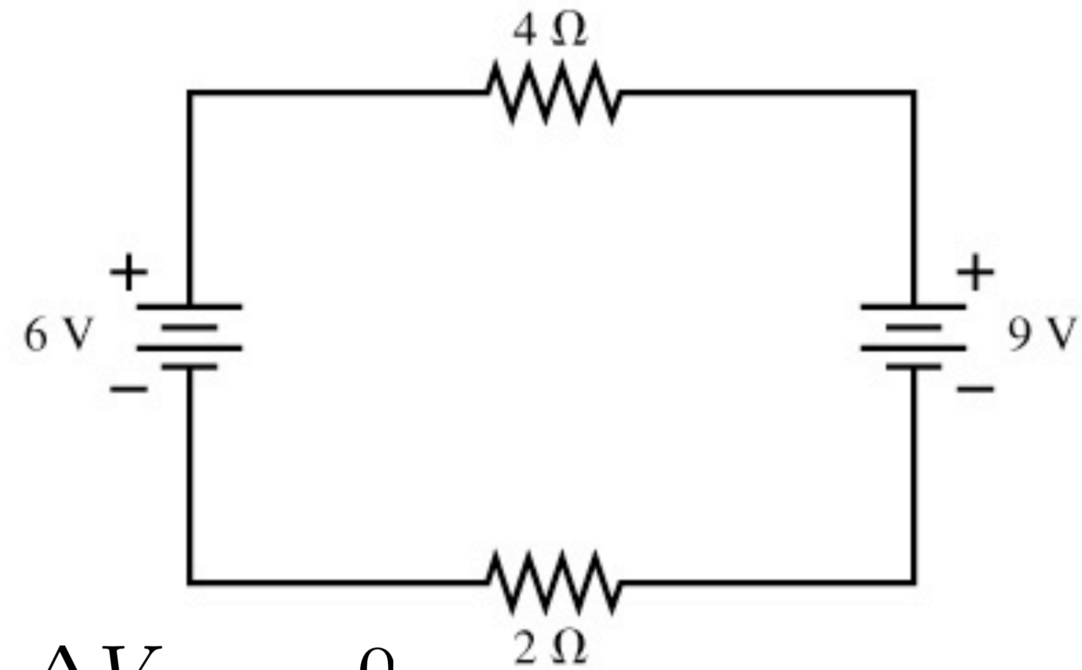
$\Delta V_R = V_{downstream} - V_{upstream} = -IR \Rightarrow$

$\mathcal{E} - IR = 0;\ I = \frac{\mathcal{E}}{R}; \Delta V_R = -IR = -\mathcal{E}$



A more complex circuit

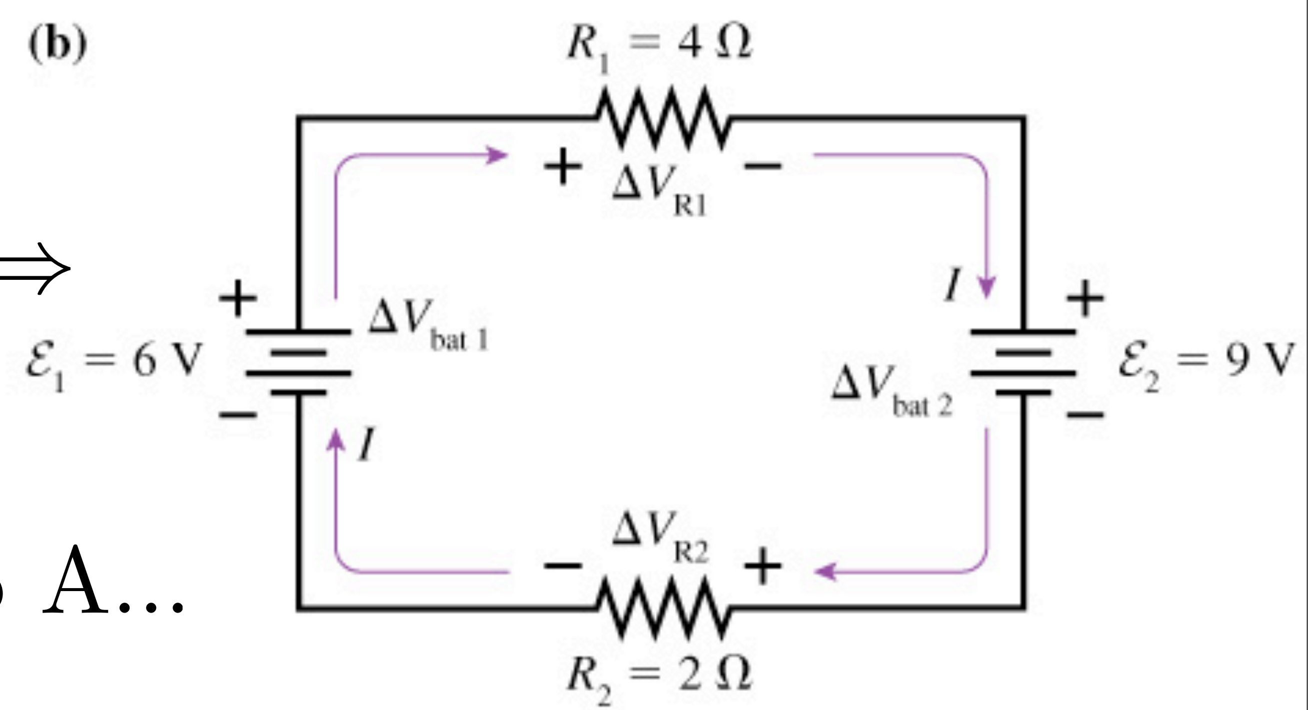
(a)



- charge can flow “backwards” thru’ battery: choose cw direction for current (if solution negative, current is really ccw)
- loop law: $\Delta V_{bat\ 1} + \Delta V_{R1} + \Delta V_{bat\ 2} + \Delta V_{R2} = 0$

$$\Delta V_{bat\ 1} = +\mathcal{E}_1; \ \Delta V_{bat\ 2} = -\mathcal{E}_2;$$

(b)



$$\Delta V_{R1} = -IR_1; \ \Delta V_{R1} = -IR_2 \Rightarrow$$

$$\mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0 \Rightarrow$$

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{6 \text{ V - } 9 \text{ V} =}{4\Omega + 2\Omega} = -0.5 \text{ A}\ldots$$

(expected: 9 V battery “dictates” direction...)

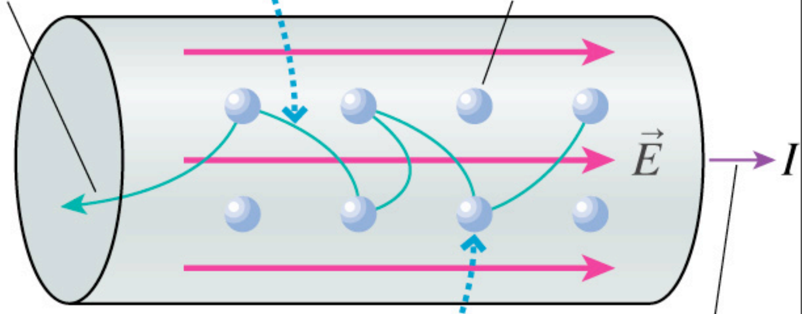
$$\Delta V_{R1} = -IR_1 = +2.0 \text{ V}\ldots$$

Energy and Power

- In battery: chemical energy potential...of charges: $\Delta U = q\Delta V_{bat} = q\mathcal{E}$
- power (rate at which energy supplied to charges): $P_{bat} = \frac{dU}{dt} = \frac{dq}{dt}\mathcal{E}$

$P_{bat} = I\mathcal{E}$ (power delivered by an emf)

- In resistor: work done on charges $qEd \longrightarrow$ kinetic (accelerate) between collisions $\longrightarrow$ thermal energy of lattice after collisions



After many collisions over length L of resistor:
$\Delta E_{th} = qEL = q\Delta V_R$

rate at which energy is transferred from current to resistor:
$P_R = \frac{dE_{th}}{dt} = \frac{dq}{dt}\Delta V_R = I\Delta V_R$

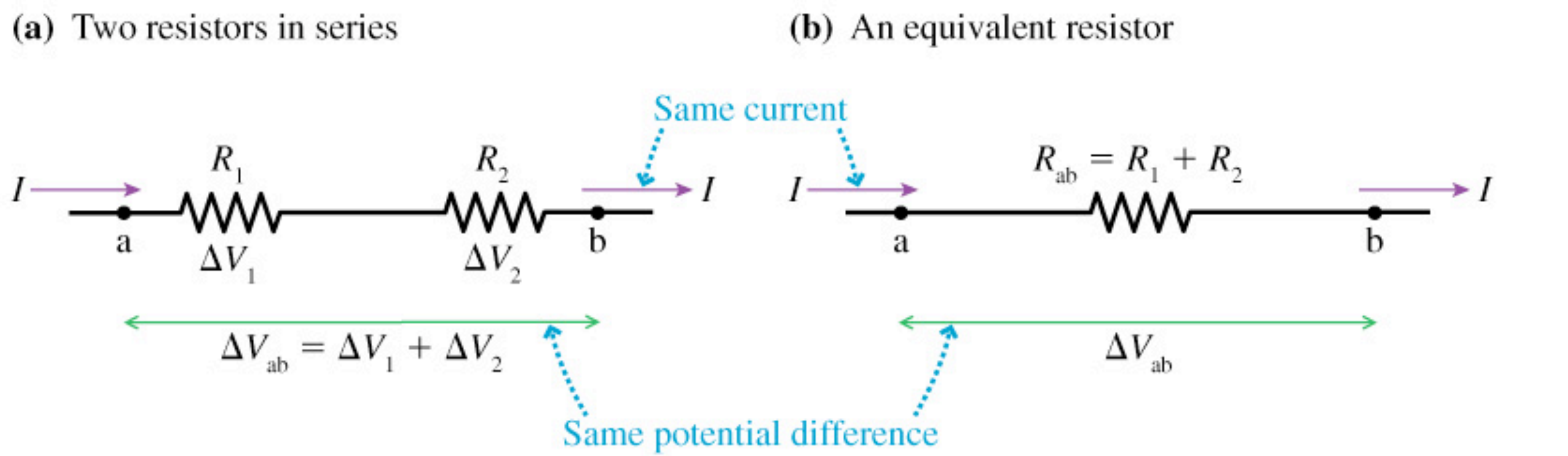
For basic circuit: $P_R = P_{bat}$ (energy conservation)

Using Ohm's law: $P_R = I\Delta V_R = I^2R = \frac{(\Delta V_R)^2}{R}$ (power dissipated by a resistor)

$E_{chem} \rightarrow U \rightarrow K \rightarrow E_{th} \rightarrow$ light...

- Common units: P_R kW in Δt hours $\rightarrow$ $P_R\Delta t$ kilowatt hours

Resistors in Series



- current same in each resistor: $\Delta V_1 = IR_1;\ \Delta V_2 = IR_2 \Rightarrow \Delta V_{ab} = I\,(R_1 + R_2)$
- equivalent resistor: $R_{ab} = \frac{\Delta V_{ab}}{I} = \frac{I(R_1+R_2)}{I} = R_1 + R_2$

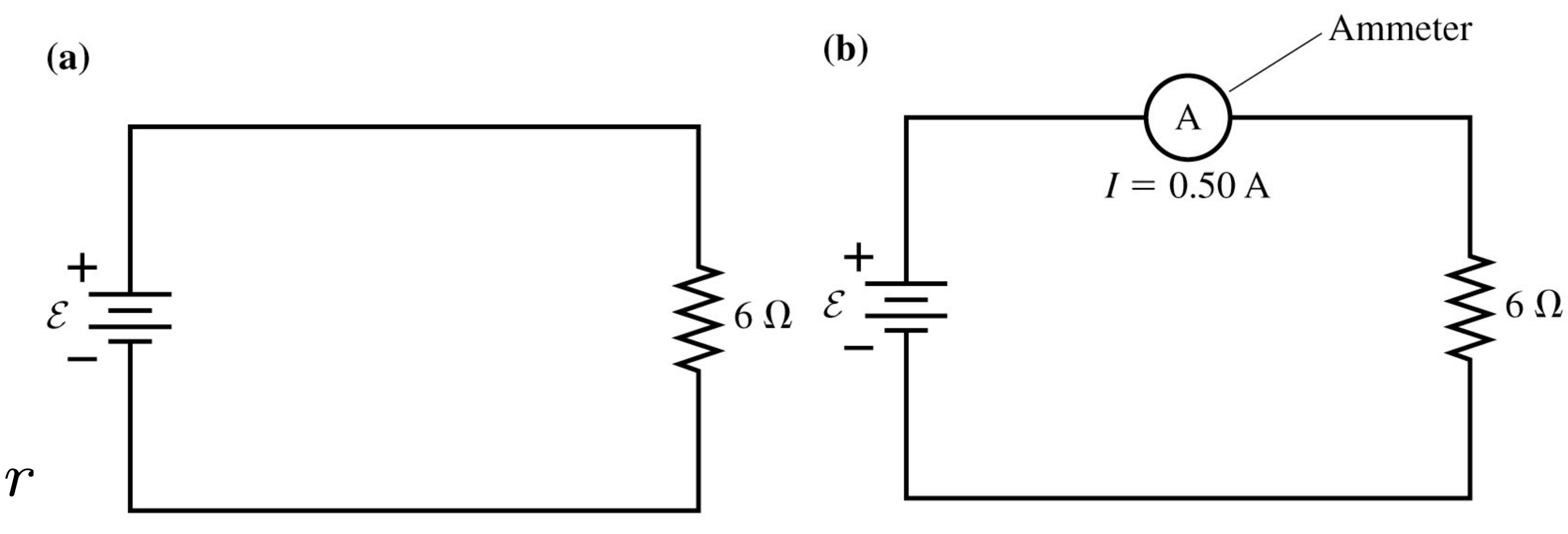
$$R_{eq} = R_1 + R_2 + \cdots + R_N \qquad \text{(series resistors)}$$

Ammeters

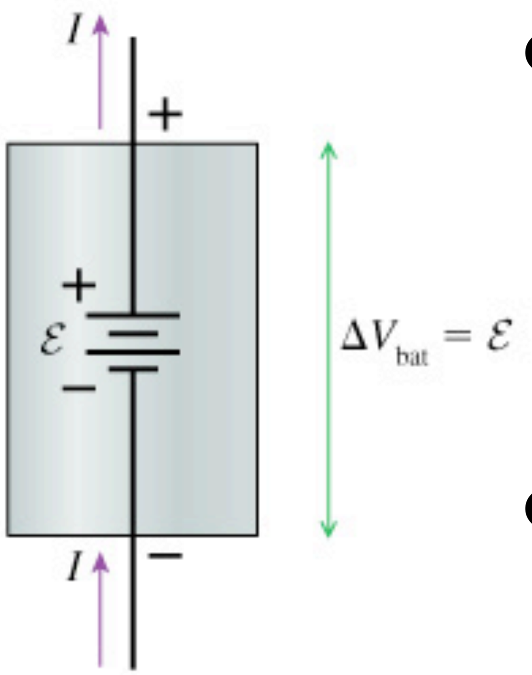
- measures current in circuit element (placed in series):

$$R_{eq} = R_{load} + R_{ammeter}$$

ideal, $R_{ammeter} = 0 \Rightarrow$ current not changed



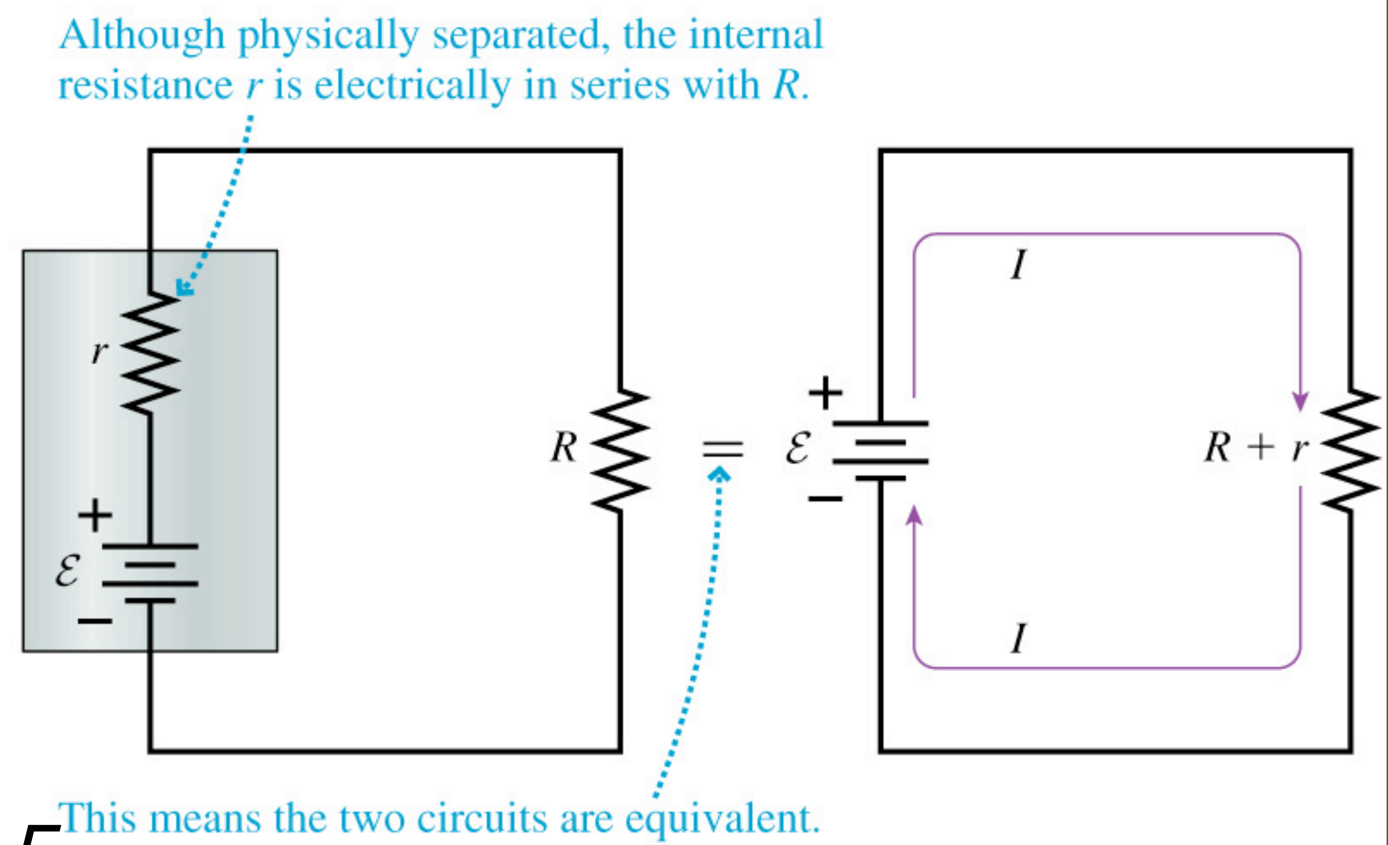
Real Batteries



Ideal battery

- terminal (user) voltage

$$\Delta V_{bat} = E - Ir$$



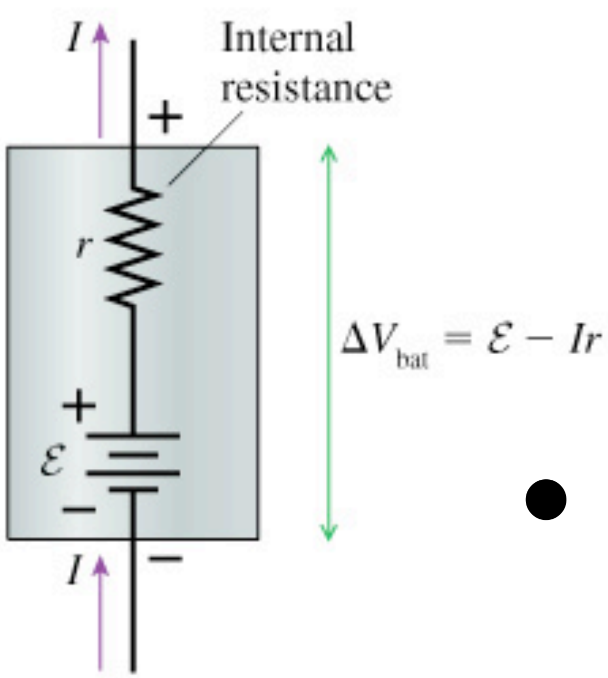
- for resistor connected:

$$I = \frac{E}{R_{eq}} = \frac{E}{R+r}$$

$$\Delta V_R = IR = \frac{R}{R+r}E$$

$$\Delta V_{bat} = E - Ir;$$

$$\Delta V_R = \Delta V_{bat} == E$$



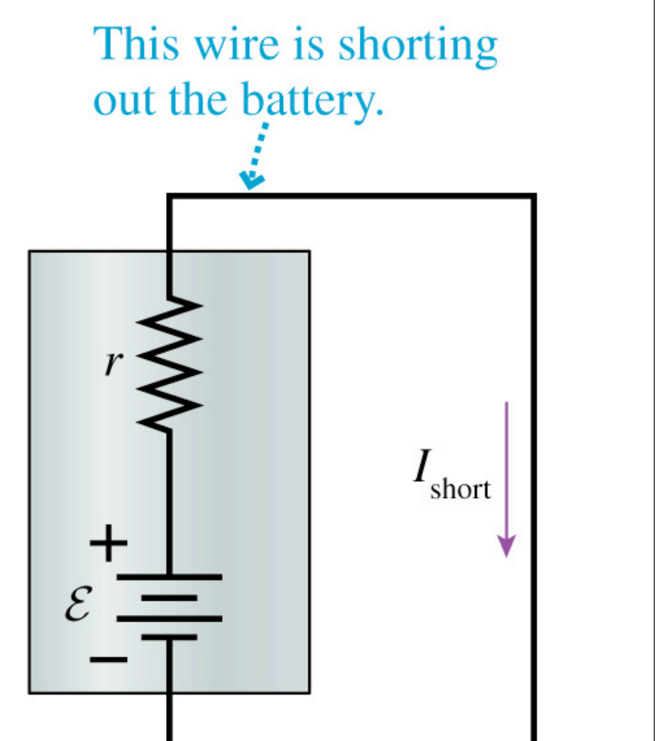
Real battery

Short Circuit

- replace high by zero resistance →

$$I_{short} = \frac{E}{r}$$

$$(\infty \text{ for ideal, } r = 0)$$



- maximum possible current battery can produce

Parallel Resistors

- potential differences same:

$$\Delta V_1 = \Delta V_2 = = V_{cd}$$

- Kirchhoff's junction law: $I = I_1 + I_2$
- Ohm's law:

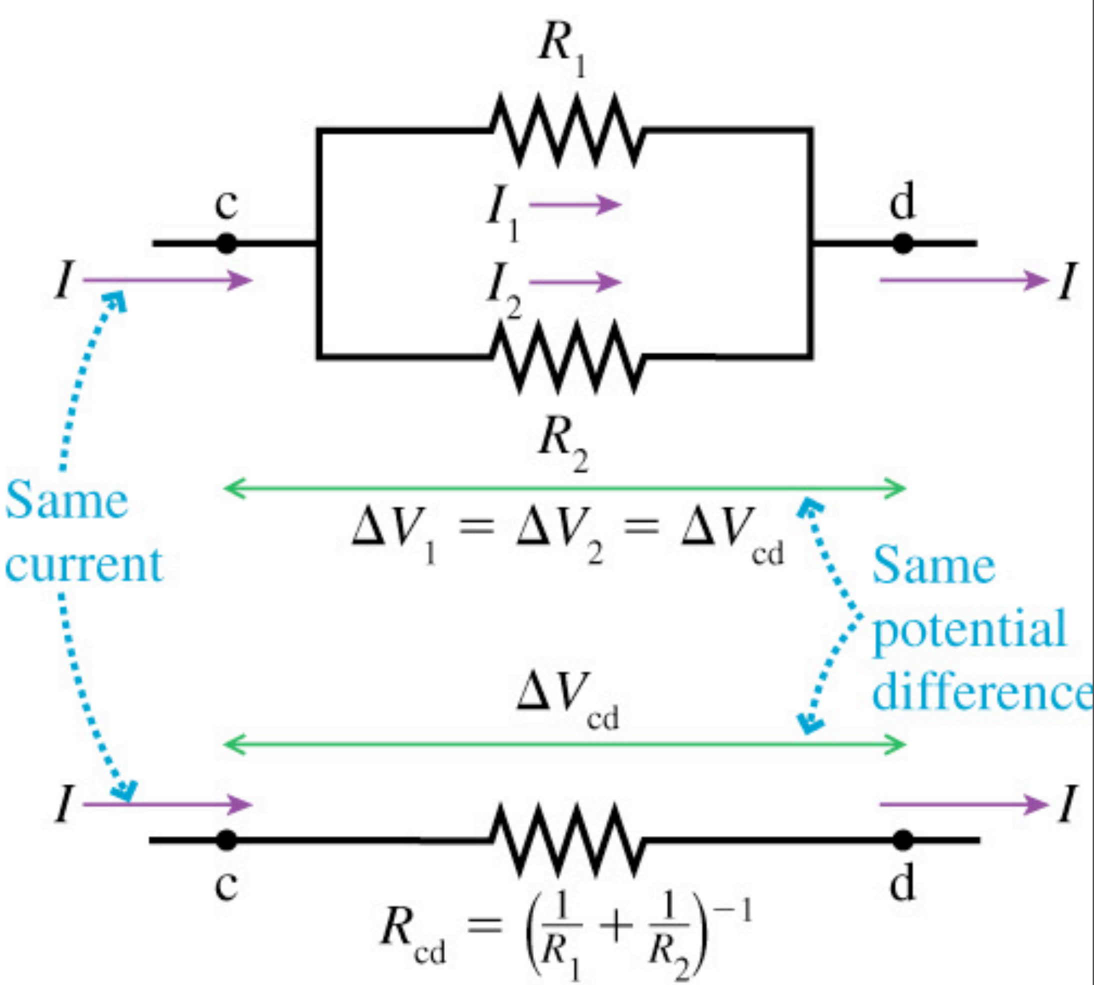
$$I = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} = \Delta V_{cd} \frac{1}{R_1} + \frac{1}{R_2}$$

- Replace by equivalent resistance:

$$R_{cd} = \frac{\Delta V_{cd}}{I} = \left[\frac{1}{R_1} + \frac{1}{R_2}\right]^{-1}$$

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right)^{-1} \qquad \text{(parallel resistors)}$$

(a) Two resistors in parallel



(b) An equivalent resistor

- Identical resistors($R_1 = R_2$): $R_{series\ eq} = 2R$; $R_{parallel\ eq} = \frac{R}{2}$
- In general, $R_{eq} < R_1$ or R_2...in parallel